$$\int_{y\ \text{constant}} 2xy\,\partial x + \int (3y^2)\text{dy=c}$$

$$2y\int_{y\ \text{constant}} x\,\partial x + 3\int (y^2)\text{dy=c} \qquad \left(\int x^n \partial x = \frac{x^{n+1}}{n+1}\right)$$

$$2y\left(\frac{x^2}{2}\right) + 3\left(\frac{y^3}{3}\right) = c$$

$$yx^2 + y^3 = c$$

Which is required solution.

Problem -02

Solve $(2ye^{2x} + 2x\cos y)dx + (e^{2x} - x^2 \sin y)dy = 0.$

Solution:-

The given DE is $(2ye^{2x} + 2x\cos y)dx + (e^{2x} - x^2 \sin y)dy = 0.$ -----(1)

Equation (1) is of the form Mdx+Ndy=0 -----(2)

Comparing the above two equations, we get

$$M = 2ye^{2x} + 2x\cos y, N = e^{2x} - x^2 \sin y$$

Differentiate M partially with respect to y (Assuming 'x' constant), we get

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} 2ye^{2x} + \frac{\partial}{\partial y} 2x\cos y$$

$$\frac{\partial M}{\partial y} = 2e^{2x}\frac{\partial}{\partial y} y + 2x\frac{\partial}{\partial y}\cos y$$

$$\frac{\partial M}{\partial y} = 2e^{2x}.1 + 2x(-\sin y) \qquad \left(\frac{\partial \cos\ ax}{\partial x} = -a\sin ax\right)$$

$$\frac{\partial M}{\partial y} = 2e^{2x} - 2x\sin y \qquad \text{----(3)}$$

Differentiate N partially with respect to x (Assuming 'y' constant), we get

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} e^{2x} - \frac{\partial}{\partial x} x^2 \sin y$$