a) The set to investigate is

$$V := \{[a, b, c] \in \mathbb{R}^3 : b = 0\}$$

This defines the set V which includes all the points from the $\mathbb{R}^3$ such that $b = 0$. Note that the zero vector is included.

$$[0, 0, 0] \in V$$

Also the additive inverse is included, because $-[a, 0, c] = [-a, 0, -c]$.

$$[a, 0, b] + \big(-[a, 0, c]\big) = [a - a, 0, c - c] = [0, 0, 0]$$

Thus V is a vector space.

b) The situtation is similar to the last one, but this time $b = 1$, which means that the zero vector cannot be in the set, so this is not a vector space.

c) First note that the set of a real numbers is a field with traditional addition and multiplication operations. This means that there is no zero divisors. Deduct from this $a = 0$ or $b = 0$. That implies the zero vector is included. To see that the additive inverse is also included take a or b to be zero and follow the reasoning from a).

d) The task here is to find out if the given vectors are linearly independent and thus span a vector space. Vectors are linearly independent if and only if

$$\alpha[1, 1, 0] + \beta[2, 0, 1] = 0$$

implies $\alpha = 0$ and $\beta = 0$. To see this is the case, just solve α and β

$$\begin{aligned} \alpha[1, 1, 0] + \beta[2, 0, 1] &= [\alpha + 2\beta, \alpha, \beta] \\ &= [0, 0, 0] \end{aligned}$$

For this to work it have to be that $\alpha = 0$ and $\beta = 0$. Thus the linear combination of given vectors span a vector space.

e) The set of vectors are now in the form

$$V := \{[a, b, c] \in \mathbb{R}^3 : c = 2b + a\}$$

The parameter c depends from a and b. A familiar consept here to think about is to rename the parameters $x = a$, $y = b$ and $z = c$ and notice that the set V is a surface over the xy-plane. When $a = b = 0$ also $c = 0$ so the zero vector is included. Because the parameter c depends linearly from a and b the additive inverse is also included.

□

Problem 6. *Are the three vectors* $[1, 2, 3]$, $[-2, 0, 1]$, *and* $[1, 10, 17]$ *linearly dependent or independent? Do they span all vectors or not?*

Solution. A set of vectors $(v)_{k=1}^{m}$ is linearly independent if and only if

$$\sum_{k=1}^{m} a_k v_k = 0$$