Systems

- A system is an abstraction of anything that takes an input signal, operates on it, and produces an output signal.
  - A system generally establishes a relationship between its input and its output.
  - Examples could be car, camera, etc.
- Systems that operate on continuous-time signal are known as continuous-time (CT) systems.
- Systems that operate on discrete-time signals are known as discrete-time (DT) systems.

Drill - 1

1. Most of the signals in this physical world is............... (CT signals / DT signals). Choose the right one.
2. Mention four systems other than those mentioned in the slides.
3. Mention three signals other than those mentioned in the slides.
4. How can we convert a CT signal into a DT signal?
5. Can a system have multiple inputs and multiple outputs?
6. What do you mean by time-domain signal and spatial-domain signal?

Three Important Cases

Case 1: Signals with finite total energy, i.e., $E_\infty < \infty$:

Such a signal must have zero average power. For example, in continuous case, if $E_\infty < \infty$, then

$$P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$$

An example of a finite-energy signal is a signal that takes on the value of 1 for $0 \le t \le 1$ and 0 otherwise. In this case, $E_\infty = 1$ and $P_\infty = 0$.

Case 2: Signals with finite average power, i.e., $P_\infty < \infty$:

For example, consider the constant signal where $x[n] = 4$. This signal has infinite energy, as

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N\to\infty} \sum_{n=-N}^{+N} 4^2 = \cdots + 16 + 16 + 16 \ldots$$

Three Important Cases - continued

However, the total average power is finite,

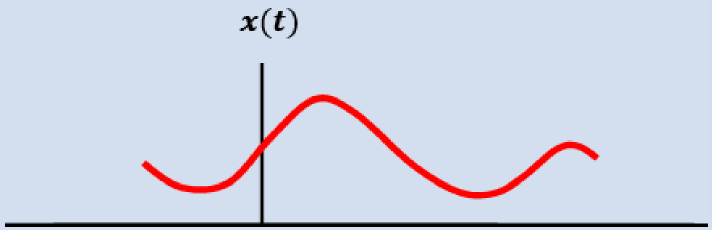
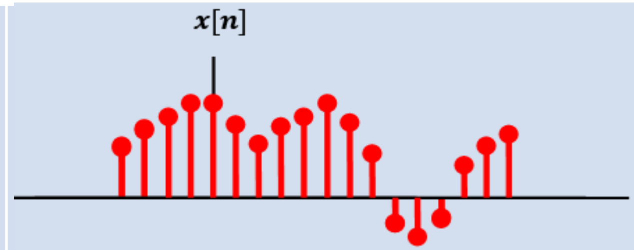
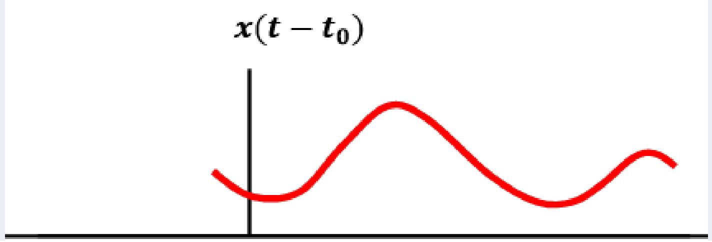
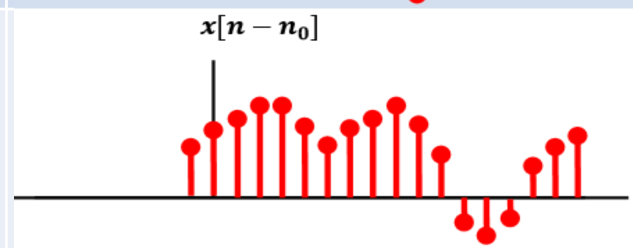
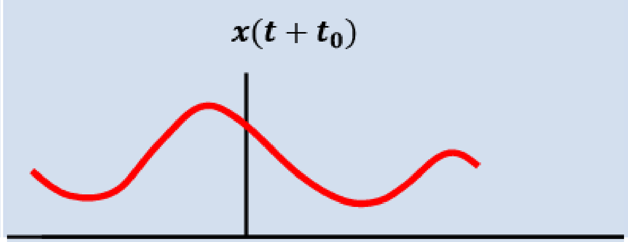
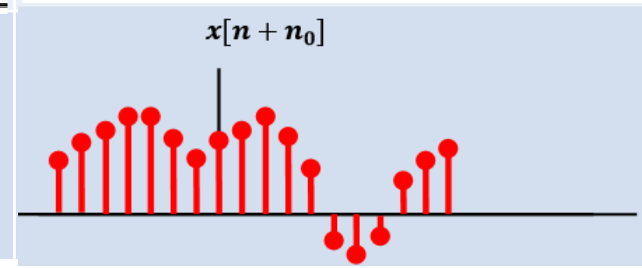
$$P_\infty \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} 4^2$$

$$= \lim_{N\to\infty} \frac{16}{2N+1} \sum_{n=-N}^{+N} 1 = \lim_{N\to\infty} \frac{16(2N+1)}{2N+1} = \lim_{N\to\infty} 16 = 16$$

Case 3: Signals with neither E_∞ nor P_∞ finite:

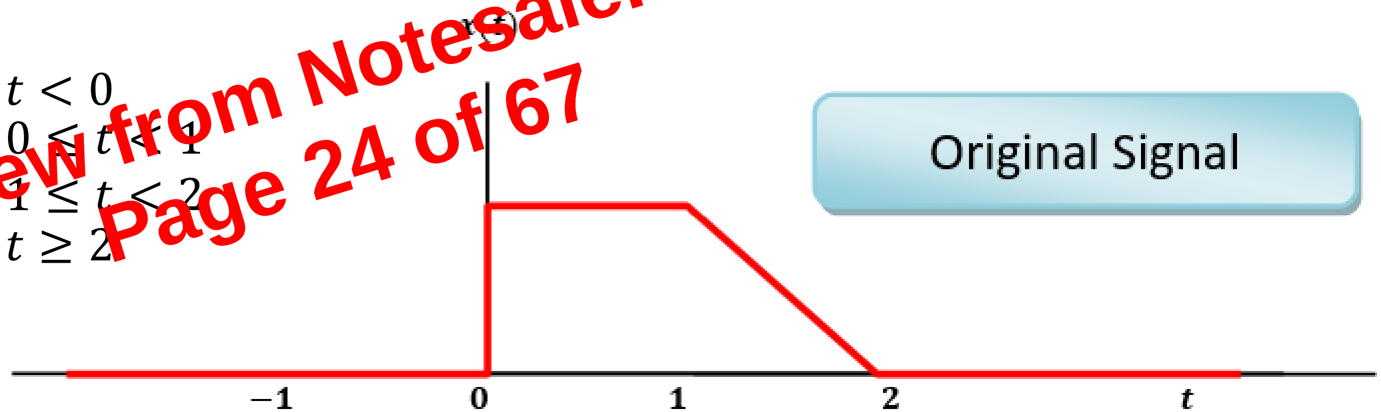
A simple example of such a case could be $x(t) = t$. In this case both E_∞ and P_∞ are infinite

Time Shift

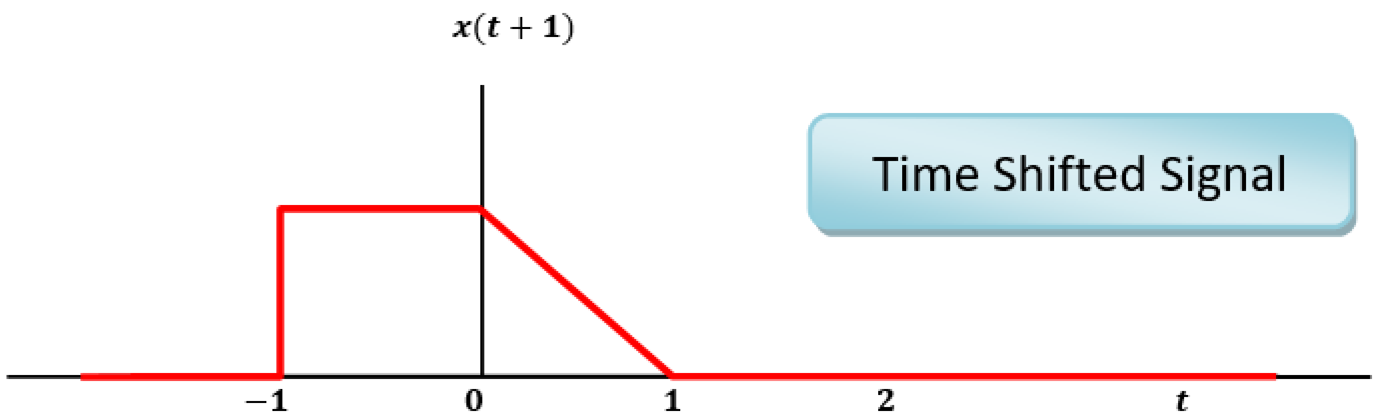
Signals	Continuous time	Discrete time
Original		
Delayed		
Advance		

Example: Time Shift: (1)

$$x(t) = \begin{cases} 0 & if\ t < 0 \\ 1 & if\ 0 \le t < 1 \\ 2 - t & if\ 1 \le t < 2 \\ 0 & if\ t \ge 2 \end{cases}$$



The signal $x(t+1)$ can be obtained by shifting the given signal to the left by one unit



Periodic Complex Exponential and Sinusoidal Signals

Now we consider the case of complex exponentials where a is purely imaginary. More, specifically, we consider:

$$x(t) = e^{j\omega_0 t}$$

An important property of this signal is that it is periodic.

$$x(t) = x(t+T) \Rightarrow e^{j\omega_0 t} = e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T} \Rightarrow e^{j\omega_0 T} = 1$$

This equation can be true,

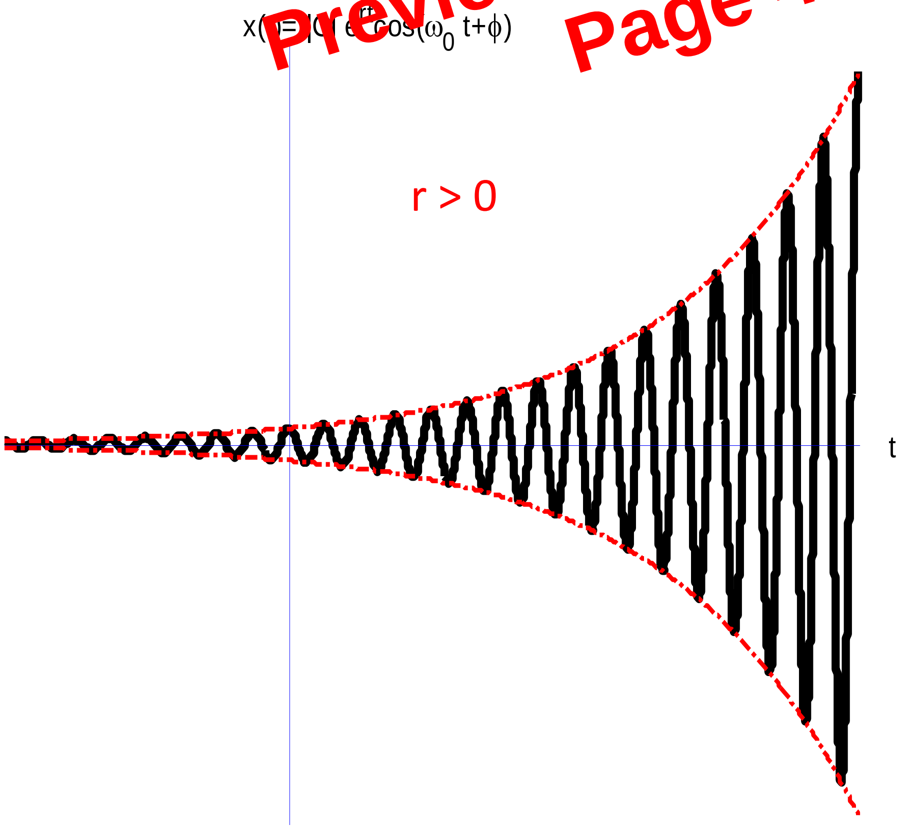
1. If, $\omega_0 = 0$, then $x(t) = 1$, which is periodic for any value of T.

2. If, $\omega_0 \neq 0$, then the fundamental period T_0 of $x(t)$, i.e. the smallest value of T for which the above equation holds, is

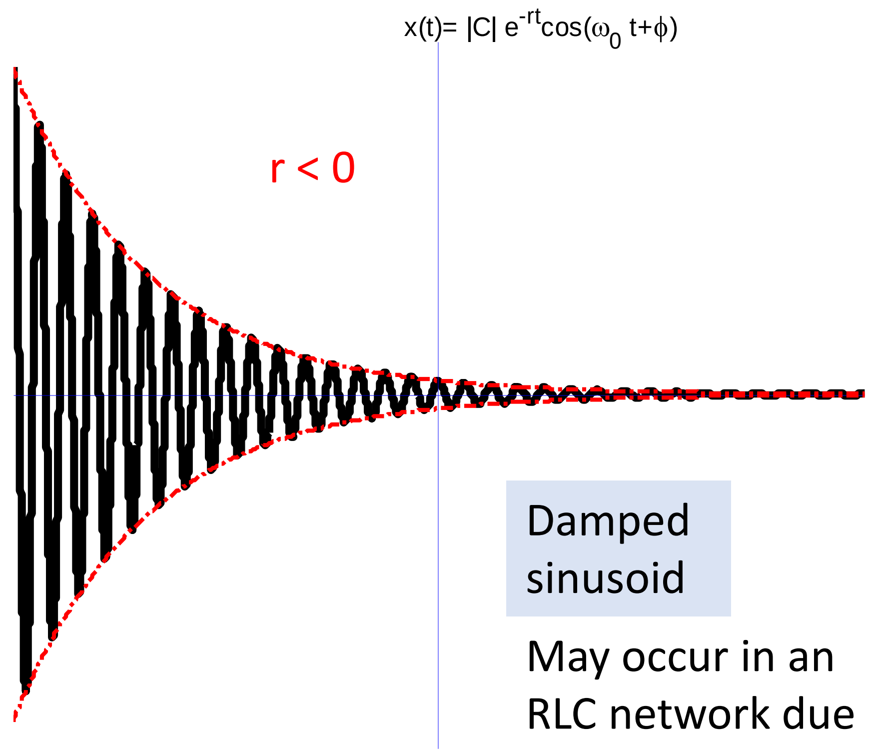
$$T_0 = \frac{2\pi}{|\omega_0|}$$

Example: General Complex Exponential Signals

Sinusoid with growing exponential



Sinusoid with decaying exponential



Damped sinusoid

May occur in an RLC network due to resistors

1.3.2 Discrete-Time Complex Exponential and Sinusoidal Signals

A discrete-time complex exponential signal or sequence $x[n]$ can be written as

$$x[n] = C\alpha^n$$

where C and α are, in general, complex numbers. This could also be written as

$$x[n] = Ce^{\beta n}$$

where $\quad \alpha = e^{\beta}$

Real Exponential Signals

In this case both C and α are real numbers, and $x[n]$ is called a real exponential.

USAGE: Real-valued discrete-time exponentials are often used to describe population growth as a function of generation, and total return on investment as a function of day, month, a quarter.

Discrete-Time Complex Exponential Signals

In case of continuous-time exponential, the signals $e^{j\omega_0 t}$ are all distinct for distinct values of ω_0.

- In discrete-time, these signals are not distinct. In fact, the signal with frequency ω_0 is identical to signals with frequencies $\omega_0 \pm 2\pi$, $\omega_0 \pm 4\pi$ and so on. Therefore, in considering discrete-time complex exponentials, we need only consider a frequency interval of size 2π. The most commonly used 2π intervals are $0 \leq \omega_0 \leq 2\pi$ or the interval $-\pi \leq \omega_0 \leq \pi$.
- As ω_0 is gradually increased, the rate of oscillations in the discrete-time signal does not keep on increasing. If ω_0 is increased from 0 to 2π, the rate of oscillations first increase and then decreases.
- Note in particular that for $\omega_0 = \pi$ or for any odd multiple of π,

$$e^{j\pi n} = \left(e^{j\pi}\right)^n = (-1)^n$$

so that the signal oscillates rapidly, changing sign at each point in time.

1.9 (a)

Workout - (8)

If the signal x(t) is periodic, find the fundamental period.

$$x(t) = je^{j10t}$$

$$= j(\cos 10t + j\sin 10t) = j\cos 10t - \sin 10t$$

$$= j\sin\left(10t + \pi/2\right) + \cos\left(10t + \pi/2\right)$$

$$= e^{\left(10t + \pi/2\right)}$$

Fundamental period:

$$T_0 = \frac{2\pi}{|\omega_0|} = \frac{2\pi}{10} = \frac{\pi}{5}$$