

1. $f(x) = x^2 - 3x$, $x \in \mathbb{R}$

(i) $f(x) > 4$

$x^2 - 3x > 4$

$x^2 - 3x - 4 > 0$

$(x-4)(x+1) > 0$

$x < -1$ or $x > 4$

(ii) $x^2 - 3x$

$= x^2 - 3x + \left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right)^2$

$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4}$

(iii) $g(x) = x - 35x$, $x \geq 0$

$g(x) = 10$

$x - 35x = 10$

$x - 35x - 10 = 0$

Let $y = 5x$

$x = y^2$

$y^2 - 3y - 10 = 0$

$(y-5)(y+2) = 0$

$y = 5$ or $y = -2$

$5x = 5$ $5x = -2$ (negative)

$x = 1$ $x = (-2)^2$

$x = 1$ $x = 4$

2. $f(x) = \frac{3}{2x+1}$, $x > 0$

$g(x) = \frac{1}{x} + 2$, $x > 0$

(i) Range of f is $0 < f(x) < 3$

Range of g is $g(x) > 2$

(ii) $fg(x) = f(g(x))$

$= f\left(\frac{1}{\frac{1}{x} + 2}\right)$

$= \frac{3}{2\left(\frac{1}{x} + 2\right) + 1}$

$= \frac{3}{\frac{2}{x} + 5}$

$= \frac{3x}{2 + 5x}$

(iii) Let $y = fg(x)$

$y = \frac{3x}{2 + 5x}$

$5xy + 2y = 3x$

$2y = 3x - 5xy$

$2y = x(3 - 5y)$

$\therefore x = \frac{2y}{3 - 5y}$

$fg^{-1}(y) = \frac{2x}{3 - 5x}$

3. (i) $xy = 12 \dots \textcircled{1}$

$2x + y = 11 \dots \textcircled{2}$

from $\textcircled{2}$ $y = 11 - 2x \dots \textcircled{3}$

Subst. $\textcircled{3}$ into $\textcircled{1}$:

$x(11 - 2x) = 12$

$2x^2 - 11x + 12 = 0$

$(2x - 3)(x - 4) = 0$

$x = \frac{3}{2}$ or $x = 4$

$\therefore y = 8$ $y = 3$

$\therefore$ The coordinates are $\left(\frac{3}{2}, 8\right)$ & $(4, 3)$

(ii) $xy = 12 \dots \textcircled{1}$

$2x + y = 11 \dots \textcircled{2}$

$y = 11 - 2x \dots \textcircled{3}$

Subst. $\textcircled{3}$ into $\textcircled{1}$

$x(11 - 2x) = 12$

$2x^2 - 11x + 12 = 0$

Does not intersect:

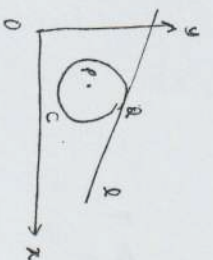
$b^2 - 4ac < 0$

$(-11)^2 - 4(2)(12) < 0$

$121 - 96 < 0$

$25 - 48 < 0$

4.



$P(7, 8)$, $Q(0, 8)$

(a) $|PA| = \sqrt{(7-10)^2 + (8-13)^2}$

$= \sqrt{34}$

(b) Equation of circle:

$(x-7)^2 + (y-8)^2 = (3\sqrt{2})^2$

$(x-7)^2 + (y-8)^2 = 36$

(c) $m(PQ) = \frac{13-8}{10-7} = \frac{5}{3}$

$\therefore m(\perp PQ) = -\frac{3}{5}$

Equation of line L:

$y - y_1 = m(x - x_1)$

$y - 13 = -\frac{3}{5}(x - 10)$

$5y - 65 = -3x + 30$

$3x + 5y + 95 = 0$

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