(D) DOUBLE ANGLES

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

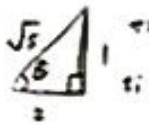
$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\cos 2A = \cos(A+A)$
$= \cos A \cos A - \sin A \sin A$
$= \cos^2 A + \sin^2 A$
$= \cos^2 A - (1-\cos^2 A)$
$= 2\cos^2 A - 1$
$= \cos^2 A - \sin^2 A$
$= (1-\sin^2 A) - \sin^2 A$
$= 1 - 2\sin^2 A$

Example:

1) Given that θ is acute and $\tan\theta = \frac{1}{2}$, evaluate each of the following:

SOH CAH TOA



$\tan\theta = \frac{1}{2}$ A

$\sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

PT: $c^2 = a^2 + b^2$, $c^2 = 1^2 + 2^2$, $c = \sqrt{5}$

(a) $\tan 2\theta$
$= \dfrac{2\tan\theta}{1-\tan^2\theta}$
$= \dfrac{2(\frac{1}{2})}{1-(\frac{1}{2})^2}$
$= \frac{4}{3}$ //

(b) $\sin 2\theta = 2\sin\theta\cos\theta$
$= 2(\frac{1}{\sqrt{5}})(\frac{2\sqrt{5}}{5})$
$= \frac{4}{5}$ //

(c) $\sec 2\theta = \dfrac{1}{\cos 2\theta}$
$= \dfrac{1}{\cos^2\theta - \sin^2\theta}$
$= \dfrac{1}{(\frac{2\sqrt{5}}{5})^2 - (\frac{\sqrt{5}}{5})^2}$
$= \frac{1}{3/5} = \frac{5}{3}$ //

2) Solve $\tan x + \tan 2x = 1$, giving all solutions in the interval $0° < x < 360°$

$\tan x + \left(\dfrac{2\tan x}{1-\tan^2 x}\right) = 1$

$2\tan x - \tan^3 x = 1 - \tan^2 x$

$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

$\therefore \tan x = \pm\sqrt{\frac{1}{3}}$

$\alpha = \tan^{-1}(\frac{1}{\sqrt{3}})$
$= 30°$

$\therefore x = 30°, 150°, 210°, 330°$