3E 2R

5 The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

V _ _ _ _ _ _ E   no restriction: $\frac{8!}{3!2!} = 3360$

No. of ways $= \frac{6!}{2!2!} = 180$

P(V as 1st letter & E as last letter) $= \frac{180}{3360}$

$= 0.053571428\overline{57}$

$= 0.0536$ (3sf)

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

$P(R_t \mid E_t) = \frac{P(R_t \cap E_t)}{P(E_t)}$

E together $E_t \cap E_t$

→ $\boxed{EEE}$ _ _ _ _ _ $\boxed{EEE}$ $\boxed{RR}$ _ _ _

$\frac{6!}{2!} = 360$ $5! = 120$

$\Rightarrow P(R_t \mid E_t) = \frac{120}{360}$

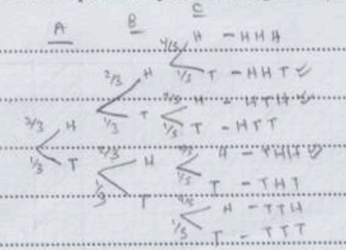
$= \frac{1}{3}$

6 Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

HHT



$P(HHT) = \left(\frac{2}{3} \times \frac{2}{3} \times \frac{1}{5}\right) + \left(\frac{2}{3} \times \frac{1}{3} \times \frac{4}{5}\right)$

$+ \left(\frac{1}{3} \times \frac{2}{3} \times \frac{4}{5}\right)$

$= \frac{4}{45} + \frac{8}{45} + \frac{8}{45}$

$= \frac{4}{9}$ (shown)

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

X – no. of heads obtained when 3 coins are thrown

x	0	1	2	3
$P(X=x)$	$\frac{1}{45}$	$\frac{8}{45}$	$\frac{4}{9}$	$\frac{16}{45}$

$P(X=0) = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{45}$

$P(X=1) = \left(\frac{1}{3} \times \frac{1}{3} \times \frac{4}{5}\right) + \left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{5}\right) + \left(\frac{1}{3} \times \frac{2}{3} \times \frac{1}{5}\right) = \frac{8}{45}$

$P(X=3) = 1 - \frac{1}{45} - \frac{8}{45} - \frac{4}{9} = \frac{16}{45}$