iii.) Find the largest value of x for which $F(x) < 1/2$

iv.) Find the smallest value of x for which $F(x) > 1/2$

v.) Find $P[0 < x < 2 / x > 0]$

i.)

x	0	1	2
$P(x)$	$3c^2$	$4c - 10c^2$	$5c - 1$

$\Sigma P(x) = 1$

$3c^2 + 4c - 10c^2 + 5c - 1 = 1$

$-7c^2 + 9c - 2 = 0$

$7c^2 - 9c + 2 = 0$

$7c^2 - 7c - 2c + 2 = 0$

$7c(c-1) - 2(c-1) = 0$

$(7c-2)(c-1) = 0$

$c = 1, \quad c = \frac{2}{7} = 0.28 \quad (\because \text{Neglect } 1)$

ii.)

x	0	1	2
$P(x)$	0.243	0.327	0.423

Find CDF $F(x)$:

x	$P(x)$	$F(x)$
0	0.243	0.243
1	0.327	0.57
2	0.423	$0.99 \simeq 1$

iii.) largest value $[F(x) < \frac{1}{2}]$

$x = 0$ largest value

iv.) smallest value $[F(x) > \frac{1}{2}]$

$x = \{1, 2\}$

$x = 1$ smallest value

5.) A probability function of infinite distribution is given by

$$P[X=j] = \frac{1}{2^j} \quad \text{for } j = 1, 2, \ldots \infty$$

i.) find the mean & variance

ii.) verify probability mass function

iii.) Probability $P[X$ is even$]$

iv.) $P[X \geq 5]$

v.) $P[X$ divisible by $3]$

Solution:

i.) $E[X] = \sum x\, P(x)$

$$= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \frac{1}{2^3} + 4 \frac{1}{2^4} + \ldots$$

$$= \frac{1}{2}\left[1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + 4 \cdot \frac{1}{2^3} + \ldots\right]$$

$= \frac{1}{4}\left[1 - \frac{1}{4}\right]^{-1}$

$= \frac{1}{4}\left[\frac{3}{4}\right]^{-1}$

$$\boxed{P[x \text{ is even}] = \frac{1}{3}}$$

iv.) $P[x \geq 5]$

$P[x \geq 5] = P(5) + P(6) + P(7) + \cdots$

$= \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \frac{1}{2^8} + \cdots$

$= \frac{1}{2^5}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots\right]$

$= \frac{1}{2^5}\left[1 - \frac{1}{2}\right]^{-1}$

$= \frac{1}{2^5}\left[\frac{1}{2}\right]^{-1}$

$= \frac{1}{16}$

v.) $P[x \text{ divisible by } 3]$

$P[x \div 3] = P(3) + P(6) + P(9) + \cdots$

$= \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \cdots$

$= \frac{1}{2^3}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \cdots\right]$

iii.) $P[0 < x < 3] = P(1) + P(2)$

$= \frac{8}{81}$

iii.) CDF :

x	P(x)	F(x)
0	1/81	1/81
1	3/81	4/81
2	5/81	9/81
3	7/81	16/81
4	9/81	[illegible]5/81
5	[illegible]/81	[illegible]6/81
6	13/81	49/81
7	15/81	64/81
8	17/81	81/81 = 1

Discrete distribution:

- Binomial
- Poisson
- Geometric
- Negative Binomial

$p = P[\text{getting 5 or 6}] = \frac{1}{6} + \frac{1}{6} = 1/3$

$p + q = 1$

$q = 1 - 1/3 \Rightarrow \boxed{q = \frac{2}{3}}$

$P[x \geq 3] = 1 - P[x < 3]$ or $P[x \geq 3] = P(3) + P(4) + P(5) + P(6)$

$= 1 - \{P(0) + P(1) + P(2)\}$

$= 1 - \{ {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{6-0} + {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{6-1} + {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 \}$

$= 1 - \{1(1)(0.087) + \cancel{6} \times \frac{1}{3} \times 0.131 + \cancel{0.6584}$

0.262

$= 0.819 \quad 1 - \{\frac{64}{729} + \frac{64}{243} + \frac{240}{729}\}$

$= \frac{233}{729}$

For 729 times $= \frac{233}{729} \times 729$

$= 233$

4.) Out of 800 families with 4 child each. how many family did you expect

i.) to have two boys and two girls

ii.) at least 1 boy

iii.) Almost 2 girls

iv.) children of both sex.