$$(u', v', w') = (\pm u, 0,0) = \pm(u, 0,0) = \pm(u, v, w).$$

Thus $\eta(u, v, w) = \eta(u', v', w')$ if and only if $(u, v, w) = \pm(u', v', w')$.

Let us prove that the image of η is a proper subset of $F^{-1}(0)$ for the map $F: \mathbf{R}^4 \to \mathbf{R}^2$ defined by

$$F(x, y, z, t) = (y(z^2 - t^2) - xzt, y^2z^2 + y^2t^2 + z^2t^2 - yzt).$$

It suffices to prove that $F \circ \eta: \mathbf{S}^2 \to \mathbf{R}^2$ a constant map equal to 0. Indeed, since $u^2 + v^2 + w^2 = 1$, we obtain that

$$\begin{aligned} F \circ \eta(u, v, w) \quad &= F(u^2 - v^2, uv, uw, vw) \\ &= (uv((uw)^2 - (vw)^2) - (u^2 - v^2)uwvw, \\ &\qquad (uv)^2(uw)^2 + (uv)^2(vw)^2 + (uw)^2(vw)^2 - uvuwvw) \\ &= (u^3vw^2 - uv^3w^2 - u^3vw^2 + uv^3w^2, \\ &\qquad u^4v^2w^2 + u^2v^4w^2 + u^2v^2w^4 - u^2v^2w^2) \\ &= (0, (v^2 + u^2 + w^2 - 1)u^2v^2w^2) = (0,0). \end{aligned}$$