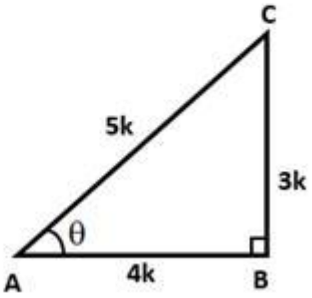


In right $\triangle ABC$, $\angle B = 90°$ and $\angle A = \theta$

4 tan θ = 3 ⇒ tan θ $\frac{3}{4} = \frac{BC}{AB}$

Let BC = 3k and AB = 4k

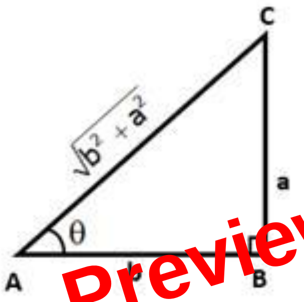
Then, $AC^2 = AB^2 + BC^2 = 16k^2 + 9k^2 = 25k^2$

$\Rightarrow AC = \sqrt{25k^2} = 5k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$ and $\cos\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$\therefore \sin\theta\cos\theta = \frac{4}{5} \times \frac{3}{5} = \frac{12}{25}$

Solution 11



In right $\triangle ABC$, $\angle B = 90°$ and $\angle A = \theta$

$\tan\theta = \frac{a}{b} = \frac{BC}{AB}$

$AC^2 = AB^2 + BC^2 = b^2 + a^2$

$$\Rightarrow AC = \sqrt{b^2 + a^2}$$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{a}{\sqrt{b^2 + a^2}} \text{ and } \cos\theta = \frac{AB}{AC} = \frac{b}{\sqrt{b^2 + a^2}}$$

$$\therefore \left(\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}\right) = \left(\frac{a \times \frac{a}{\sqrt{b^2+a^2}} - b \times \frac{b}{\sqrt{b^2+a^2}}}{a \times \frac{a}{\sqrt{b^2+a^2}} + b \times \frac{b}{\sqrt{b^2+a^2}}}\right)$$

$$= \left(\frac{\frac{a^2}{\sqrt{b^2+a^2}} - \frac{b^2}{\sqrt{b^2+a^2}}}{\frac{a^2}{\sqrt{b^2+a^2}} + \frac{b^2}{\sqrt{b^2+a^2}}}\right)$$

$$= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)$$

Solution 12

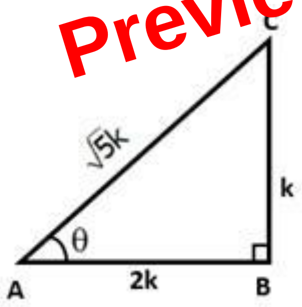
$\sin\theta = \frac{12}{13}$ ....given

We know that,

$$\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\therefore \left(\frac{2\sin\theta - 3\cos\theta}{4\sin\theta + 9\cos\theta}\right) = \left(\frac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} + 9 \times \frac{5}{13}}\right) = \left(\frac{24 - 15}{48 - 45}\right) = \frac{9}{3} = 3$$

Solution 13



$$\tan\theta = \frac{1}{2} = \frac{BC}{AB}$$

In right $\triangle ABC$, $\angle B = 90°$ and $\angle A = \theta$

Let BC = k and AB = 2k

Then, $AC^2 = AB^2 + BC^2 = (4k^2 + k^2) = 5k^2$

Consider ΔABC where $\angle B = 90^\circ, \angle A = \theta$

$$\text{Then, } \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{a^2 - b^2}{a^2 + b^2}$$

Let $BC = a^2 - b^2$ and $AC = a^2 + b^2$

Then, by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2 = a^4 + b^4 + 2a^2b^2 - a^4 - b^4 + 2a^2b^2 = 4a^2b^2$

$\Rightarrow AB = 2ab$

Now,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{2ab}{a^2 + b^2}$$

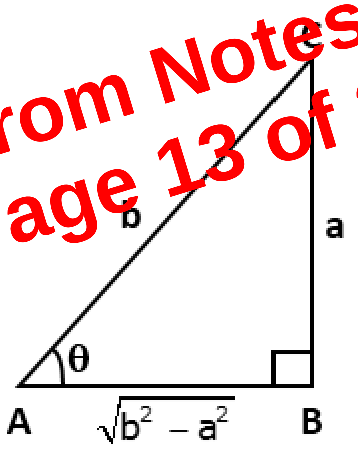
$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{a^2 - b^2}{2ab}$$

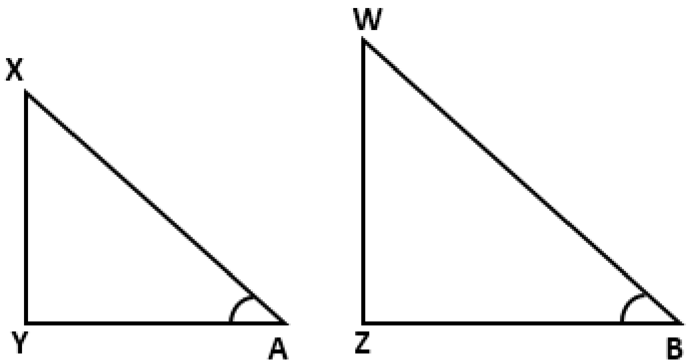
$$\cot\theta = \frac{1}{\tan\theta} = \frac{2ab}{a^2 - b^2}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{a^2 + b^2}{a^2 - b^2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{a^2 + b^2}{2ab}$$

Solution 10





We have,

$$\tan A = \frac{XY}{AY} \text{ and } \tan B = \frac{WZ}{BZ}$$

Since tan A = tan B

$$\Rightarrow \frac{XY}{AY} = \frac{WZ}{BZ}$$

$$\Rightarrow \frac{XY}{WZ} = \frac{AY}{BZ} = k\,(\text{say}) \quad \ldots.(i)$$

$$\Rightarrow XY = k \times WZ \text{ and } AY = k \times BZ \quad \ldots.(ii)$$

Using Pythagoras theorem in triangles XAY and WBZ, we have

$$XA^2 = XY^2 + AY^2 \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow XA^2 = k^2WZ^2 + k^2BZ^2 \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow XA^2 = k^2(WZ^2 + BZ^2) \quad \text{and} \quad WB^2 = WZ^2 + BZ^2$$

$$\Rightarrow \frac{XA^2}{WB^2} = \frac{k^2(WZ^2 + BZ^2)}{(WZ^2 + BZ^2)} = k^2$$

$$\Rightarrow \frac{XA}{WB} = k \quad \ldots.(iii)$$

From (i), (ii) and (iii), we get

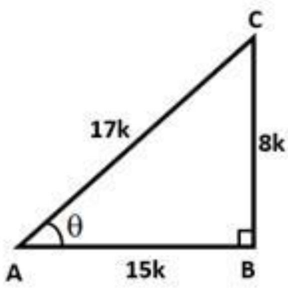
$$\frac{XY}{WZ} = \frac{AY}{BZ} = \frac{XA}{WB}$$

$$\Rightarrow \Delta AYX \sim \Delta BZW$$

$$\Rightarrow \angle A = \angle B$$

Trigonometric Ratios Exercise MCQ

Solution 1



$$\tan\theta = \frac{BC}{AB} = \frac{8}{15}$$