

⑥

$$\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & ; xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & ; x > 0, y > 0 \text{ and } xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & ; x < 0, y < 0 \text{ and } xy > 1 \end{cases}$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right) ; x > 0, y > 0 \text{ and } xy < 1$$

⑦ $\sin^{-1}x \pm \sin^{-1}y$
 $\cos^{-1}x \pm \cos^{-1}y$ } Convert each $\sin^{-1}x, \sin^{-1}y, \cos^{-1}x$ and $\cos^{-1}y$ into $\tan^{-1}x$ and $\tan^{-1}y$ by drawing triangle and then apply property ⑥

eg:- $\cos^{-1}\frac{2}{3} \Rightarrow$  $\Rightarrow \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ i.e. $\cos^{-1}\frac{2}{3} = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

⑧

$$2\tan^{-1}x = \begin{cases} \sin^{-1}\left(\frac{2x}{1+x^2}\right) & ; |x| \leq 1 \\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) & ; x \geq 0 \\ \tan^{-1}\left(\frac{2x}{1-x^2}\right) & ; x \in (-1, 1) \end{cases}$$

⑨

$$\sin^{-1}(\sin 2\theta) = \begin{cases} -\pi - 2\theta & ; \theta \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \\ 2\theta & ; \theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \\ \pi - 2\theta & ; \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \end{cases}$$

Put $x = \tan \theta$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} -\pi - 2\tan^{-1}x & ; x < -1 \\ 2\tan^{-1}x & ; x \in [-1, 1] \\ \pi - 2\tan^{-1}x & ; x > 1 \end{cases}$$

⑩

$$\cos^{-1}(\cos 2\theta) = \begin{cases} 2\theta & ; x \in [0, \infty) \\ -2\theta & ; x \in (-\infty, 0) \end{cases}$$

Put $x = \tan \theta$

$$\cos^{-1}(\cos 2\theta) = \begin{cases} 2\tan^{-1}x & ; x \geq 0 \\ -2\tan^{-1}x & ; x < 0 \end{cases}$$

⑪

$$\tan^{-1}(\tan 2\theta) = \begin{cases} \pi + 2\theta & ; \theta \in \left(-\frac{\pi}{2}, -\frac{\pi}{4}\right) \\ 2\theta & ; \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \\ -\pi + 2\theta & ; \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \end{cases}$$

Put $x = \tan \theta$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} \pi + 2\tan^{-1}x & ; x < -1 \\ 2\tan^{-1}x & ; x \in (-1, 1) \\ -\pi + 2\tan^{-1}x & ; x > 1 \end{cases}$$