Exercises II

Evaluate the following expressions without a calculator.

(a) 8×27

(b) 14×7

(c) 14×27

(d) 89×76

(e) $(a+b+c) \times (d+e+f)$

Factorization

Factoring is the opposite of distributing. That is, factoring breaks down an expression into the product of its simplest components.

[redacted]

Examples

1. The number 21 can be written as $21 = 3 \times 7$. We say: "3 and 7 are factors of 21". Because 3 and 7 are both prime numbers we call this a *prime factorization*.

2. The number 48 can be written as $48 = 4 \times 12$. Both 4 and 12 are factors of 48, but neither are prime numbers, so we must find the prime factorizations of those numbers first.

$$4 = 2 \times 2 \qquad 12 = 2 \times 6$$
$$= 2 \times 2 \times 3$$

   Notice that 2 and 3 are both prime numbers. Thus the prime factorization of 48 is $48 = 2 \times 2 \times 2 \times 2 \times 3$ or $48 = 2^4 \times 3$

3. You can factor a common divisor out of a sum: $ab + ac = a(b + c)$.
   Consider the sum $6 + 10$.
   6 and 10 are both even numbers and therefore divisible by 2. Thus:

$$6 + 10 = 2 \times 3 + 2 \times 5$$
$$= 2(3 + 5)$$

Exercises III

Find the prime factorizations of the following integers.

(a) 35

(b) 36

(c) 144

Factor a common divisor out of the following sums.

(d) $14 + 63 + 35$

(e) $6 + 54 + 12 + 48 + 18 + 42 + 24 + 36 + 30$

Solving Equations

In ten years, Matt will be twice as old as he was six years ago. How old is he right now?

How can you solve this problem?

You *could* "guess and check", where you start guessing numbers and check to see if they satisfy the conditions. But this is inefficient; it takes too long and doesn't work with more complicated problems.

Instead, you can set up an equation with a variable representing Matt's age right now. A **variable** is a symbol that represents an unknown quantity. In algebra it is often our goal to isolate a variable so that it is no longer unknown.

Steps for solving:

1. Determine what you are trying to isolate/solve for.

2. Simplify the equation as much as possible by adding and subtracting like terms.

   Like terms are terms in a mathematical equation that have the exact same variables; only their coefficients are different.

   You can think of it like adding apples and oranges. If I have 3 apples plus 2 apples plus 5 oranges plus 1 orange, I actually have 5 apples and 6 oranges. Another, more mathematical, example:

   $$5 + x + 3y - 2 - y + 2x = 3 + 3x + 2y$$

3. Isolate the desired variable on one side of the equal sign and everything else on the other side by performing opposite operations *in reverse BEDMAS order*. The goal of isolating a variable, say x, is to obtain the form $x = ...$ or $... = x$. Notice that x is positive with a coefficient of 1. There should be no other xs on the other side of the equal sign.