ou

$$z = r(\cos\theta + i\sin\theta), \ \ forme\ trigonomtrique\ du\ nombre\ complexe\ z,$$

de plus

$$z = x + iy \ dite\ forme\ algbrique\ de\ z.$$

Exemple 1.2. *Donner la forme polaire de* $z = 1 + i$.
On a $r = \mid z \mid = \sqrt{2}$ *et*

$$\begin{cases} \cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}}, \\ \sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}}, \end{cases} \quad \Rightarrow \theta = arg(z) = \frac{\pi}{4}.$$

Ainsi la forme exponentielle est

$$z = re^{i\theta} = \sqrt{2}e^{i\frac{\pi}{4}},$$

et la forme trigonomértique est

$$z = r(\cos(\theta) + i\sin(\theta)) = \sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})).$$

- **Formule de De Moivre :**

$$si \ \ z = re^{i\theta} = r(\cos(\theta) + i\sin(\theta)), \ \ alors \ \ z^n = r^n e^{in\theta} = r^n(\cos(n\theta) + i\sin(n\theta)).$$

Exemple 1.3.

$$\begin{aligned} (\frac{\sqrt{3}}{2} + \frac{i}{2})^3 &= (re^{i\theta})^3 = (e^{i\frac{\pi}{6}})^3 = e^{i3\frac{\pi}{6}} \\ &= (\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6}))^3 = (\cos(3\frac{\pi}{6}) + i\sin(3\frac{\pi}{6})) \\ &= \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2}) = i \end{aligned}$$

- **Racines $n^{ème}$ d'un nombre complexe :**

$$si \ \ z^n = r(\cos(\theta) + i\sin(\theta)),$$

$$alors \ \ z = r^{\frac{1}{n}}(\cos(\frac{\theta + 2k\pi}{n}) + i\sin(\frac{\theta + 2k\pi}{n})), \ \ k = 0, 1, 2, \ldots, (n-1).$$

Exemple 1.4. *Les racines cubiques de l'unité,* $z^3 = 1$,

$$\begin{aligned} & z^3 = 1 = 1e^{i0} = 1(\cos(0) + i\sin(0)) \\ \Rightarrow \ & z_k = 1^{\frac{1}{3}}(\cos(\frac{0 + 2k\pi}{3}) + i\sin(\frac{0 + 2k\pi}{3})), \ \ k = 0, 1, 2 \\ \Rightarrow \ & z_k = \cos(\frac{2k\pi}{3}) + i\sin(\frac{2k\pi}{3}), \ \ k = 0, 1, 2. \end{aligned}$$

Ainsi les racines cubiques de 1 sont

$$\begin{aligned} k = 0 \ \ &\Rightarrow \ \ z_0 = \cos(0) + i\sin(0) = 1, \\ k = 1 \ \ &\Rightarrow \ \ z_1 = \cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3}) = \frac{-1}{2} + i\frac{\sqrt{3}}{2}, \\ k = 2 \ \ &\Rightarrow \ \ z_2 = \cos(\frac{4\pi}{3}) + i\sin(\frac{4\pi}{3}) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}. \end{aligned}$$