

- (a) Only one mode of transport
- (b) Two different mode of transport.

SEQUENCE AND SERIES

2016 OCT/NOV

1. The first three terms of a geometric progression are $x + 1$, $x - 3$ and $x - 1$.
Find;
- (a) The value of x
 - (b) The first term
 - (c) The sum to infinity.

2017 GCE

2. The first three terms of a geometric progression are $6 + n$, $10 + n$ and $15 + n$.
Find;
- (a) The value of n
 - (b) The common ratio
 - (c) The sum of the first 6 terms of this sequence.

2017 INTERNAL

3. For the geometric progression $20, 5, 1\frac{1}{4}, \dots$, find
- (a) The common ratio,
 - (b) The n^{th} term,
 - (c) The sum of the first 8 terms.

2018 GCE

4. In a geometric progression, the third term is $\frac{2}{9}$ and the fourth term is $\frac{2}{27}$.
Find ;
- (a) The first term and the common ratio,
 - (b) The sum of the first 5 terms,
 - (c) The sum to infinity.

2018 INTERNAL

2010 Oct/Nov Exams

7. If $P = \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$, find;

- (i) PQ (ii) the value of a and b given that $PQ = P - Q$

2011 Oct/Nov Exams

8. Given that matrix $A = \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$.

- (i) Write an expression in terms of x for the determinant of A .
 (ii) Find the value of x , given that the determinant of A is 5.
 (iii) Write A^{-1}

2012 Oct/Nov Exams

9. Given that $A = \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, find the

- (i) Determinant of A (ii) A^{-1} (iii) value of AB

2013 Oct/Nov Exams

10. Given that $A = \begin{pmatrix} 2 & 2 & -1 \end{pmatrix}$, $P = \begin{pmatrix} -1 & -1 \\ 4 & 2 \\ 2 & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$, find;

- (i) $2P$ (ii) determinant of Q (iii) AP

2014 Oct/Nov Exams

11. Given that $A = \begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, find;

- (i) A^{-1} (ii) $3A - B$ (iii) AB

2015 Oct/Nov Exams

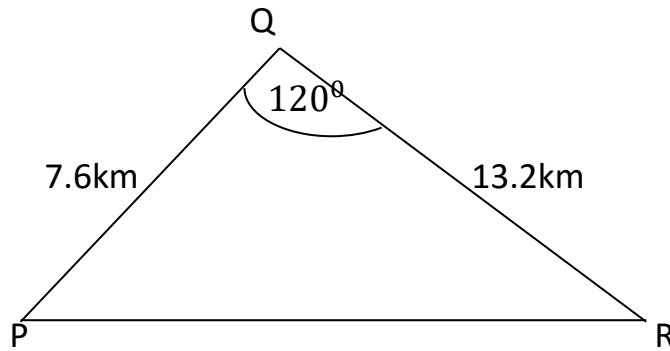
12. Given that matrix $Q = \begin{pmatrix} a & 2 \\ 3 & -2 \end{pmatrix}$.

- (i) Write an expression in terms of a , for the determinant of Q .
 (ii) Find the value of a , given that the determinant of Q is 2.
 (iii) Write Q^{-1}

2016 Oct/NOV Exams

13. Given that $Q = \begin{pmatrix} 3 & -2 \\ x & 4 \end{pmatrix}$, find

- (i) the value of x , given that the determinant of Q is 2



(a) Calculate

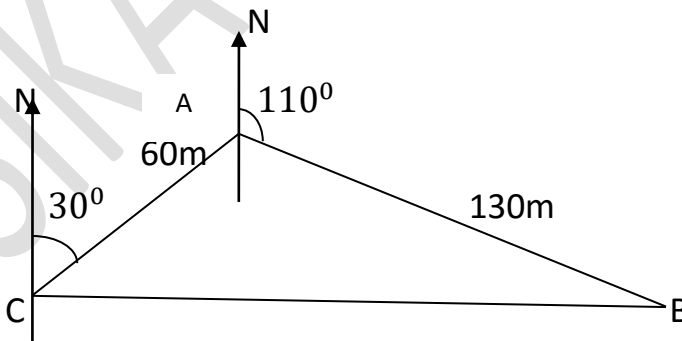
(i) The distance PR

(ii) The area of triangle PQR

(b) Find the shortest distance from Q to PR.

(c) A fisherman takes 30 minutes to move from R to P. Calculate his average speed in km/h.

5. A girls' high school has been built in such a way that the Administration block (A), dormitories (B) and classes (C) are connected by straight corridors. A is 60m from C and 130m from B. The bearing of B from A is 110° and the bearing of A from C is 030° as shown in the diagram below.



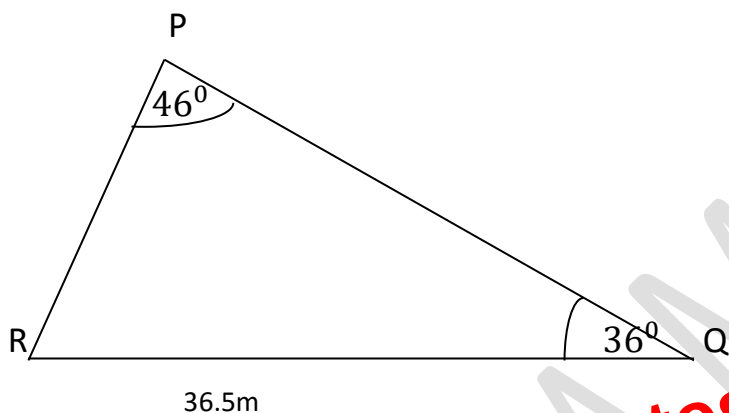
(i) Find angle BAC

(ii) Calculate the distance BC

(b) Solve the equation $\tan \theta = 0.7$ for $0^\circ \leq \theta \leq 180^\circ$.

2017 GCE Exams

11. (a) In the triangle PQR below, $QR = 36.5\text{m}$, angle $PQR = 36^\circ$ and angle $QPR = 46^\circ$.



Calculate

- (i) PQ
- (ii) the area of triangle PQR
- (iii) the shortest distance from R to PQ.

2017 Oct/Nov Exams

12. (a) The diagram below shows the location of houses for a village Headman (H), his secretary (S) and a trustee (T). H is 1.3km from S. T is 1.9km from H and angle $THS = 130^\circ$.

$$=3[(y+2)(y-2)]$$

$$\frac{ax-3a}{(x-1)(x-2)}$$

2014

13. (a) $5x - 8 - 3(x + 1) = -7$

(b) $5(2y - 3) - 2(5 - 2y)$

(c) $\frac{5}{2y-1} - \frac{6}{3y-1}$

$$5x - 8 - 3x - 3 = -7$$

$$=10y - 15 - 10 + 4y$$

$$\frac{5(3y-1)-6(2y-1)}{(2y-1)(3y-1)}$$

$$5x - 3x - 8 - 3 = -7$$

$$=10y + 4y - 25$$

$$\frac{15y-5-12y+6}{(2y-1)(3y-1)}$$

$$2x - 11 = -7$$

$$=14y - 25$$

$$\frac{15y-12y-5+6}{(2y-1)(3y-1)}$$

$$2x = -7 + 11$$

$$\frac{3y+1}{(2y-1)(3y-1)}$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

(d) $9t - 4 < 12t - 10$

$$9t - 12t < -10 + 4$$

$$\frac{-3t}{-3} < \frac{-6}{-3}$$

$$t > 2$$

(e) $2xy + 8x - 3y - 12$

$$2x(y + 4) - 3(y + 4)$$

$$=(2x - 3)(y + 4)$$

2015

14.(a) $5px - 5py + 3qx - 3qy$

(b) $\frac{2y^2 - 3y - 5}{y^2 - 1}$

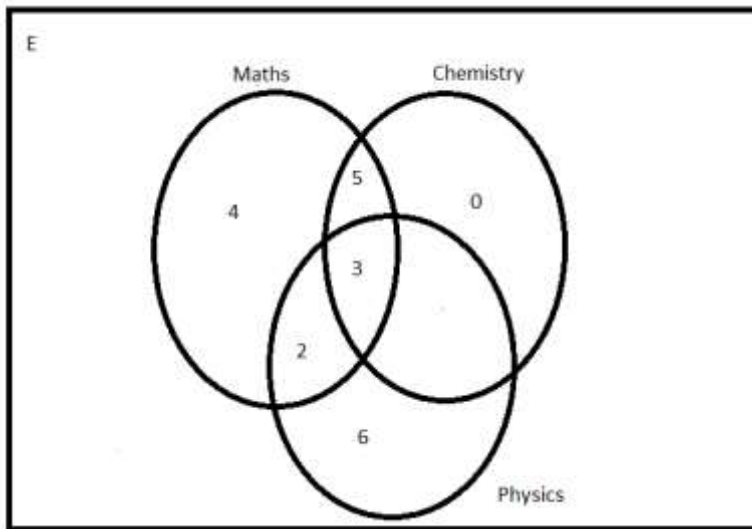
prod=-10 sum=-3 factors=-5,2

$$=5p(x - y) + 3q(x - y)$$

$$= \frac{2y^2 + 2y - 5y - 5}{y^2 - 1}$$

$$=(5p + 3q)(x - y)$$

$$= \frac{2y(y+1) - 5(y+1)}{y^2 - 1^2}$$

2018GCE

14.(i)

(ii) Physics only = **6 students**(b) two subjects only = $2 + 5 = 7$ (c) Maths and Physics not Chemistry = **2****2018 Oct/Nov**15.(i) $4 + x + 3 + 7 = 22$

$$x + 14 = 22$$

$$x = 22 - 14$$

$$x = 8$$

(ii) (a) one mode of transport = $7 + 14 + 7 = 28$

(b) two different mode of transport

$$= 4 + 2 + 3 + 8$$

$$= 17$$

SEQUENCE AND SERIES**2016 INTERNAL**1. (a) To find the common ratio, we use $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$\frac{x-3}{x+1} = \frac{x-1}{x-3}$$

$$(x-3)(x-3) = (x+1)(x-1)$$

$$x^2 - 3x - 3x + 9 = x^2 - x + x - 1$$

$$(c) S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{\frac{8}{3}}{1 - (-\frac{1}{2})}$$

$$S_{\infty} = \frac{\frac{8}{3}}{1 + \frac{1}{2}}$$

$$\text{Gradient of the normal} = \frac{-1}{\frac{dy}{dx}}$$

$$= -\frac{1}{6}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{6}(x - 2)$$

$$y - 2 = -\frac{1}{6}x + \frac{1}{3}$$

$$y = -\frac{1}{6}x + \frac{1}{3} + 2$$

$$y = \frac{-x+2+12}{6}$$

$$\frac{y}{1} = \frac{-x+14}{6}$$

$$6y = -x + 14$$

$$(b) \text{ At the stationary point, } \frac{dy}{dx} = 0$$

$$3x^2 - 3x = 0$$

$$3x(x - 1) = 0$$

$$3x = 0 \text{ or } x - 1 = 0$$

$$x = 0 \text{ or } x = 1$$

$$\text{when } x = 0, y = (0)^3 - \frac{3}{2}(0)^2$$

$$y = 0$$

$$\text{when } x = 1, y = (1)^3 - \frac{3}{2}(1)^2$$

$$y = \frac{2-3}{2} = -\frac{1}{2}$$

$\therefore$ the stationary points are:

$$(0, 0) \text{ and } (1, -\frac{1}{2})$$

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MATRICES

2002

$$\begin{aligned} 1. (i) MN &= \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \times 3 + 5 \times -1 & 2 \times -5 + 5 \times 2 \\ 1 \times 3 + 3 \times -1 & 1 \times -5 + 3 \times 2 \end{pmatrix} \\ MN &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} (ii) NM &= \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 \times 2 + (-5) \times 1 & 3 \times 5 + (-5) \times 3 \\ -1 \times 2 + 2 \times 1 & -1 \times 5 + 2 \times 3 \end{pmatrix} \\ NM &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

2003

$$\begin{aligned} 2. (i) B - 2A &= \begin{pmatrix} 6 & 2 \\ 4 & 0 \end{pmatrix} - 2 \begin{pmatrix} 2 & 1 \\ -5 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 2 \\ 4 & 0 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ -10 & -6 \end{pmatrix} \\ &= \begin{pmatrix} 6-4 & 2-2 \\ 4+10 & 0+6 \end{pmatrix} \\ B - 2A &= \begin{pmatrix} 2 & 0 \\ 14 & 6 \end{pmatrix} \end{aligned}$$

$$(ii) BA^{-1}$$

$$\text{deter } A = (2 \times -3) - (-5 \times 1)$$

$$= -6 + 5$$

$$= -1$$

$$A^{-1} = \frac{1}{-1} \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ -5 & -2 \end{pmatrix}$$

Therefore, $BA^{-1} = \begin{pmatrix} 6 & 2 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -5 & -2 \end{pmatrix}$

$$= \begin{pmatrix} 6 \times 3 + 2 \times -5 & 6 \times 1 + 2 \times -2 \\ 4 \times 3 + 0 \times -5 & 4 \times 1 + 0 \times -2 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 2 \\ 12 & 4 \end{pmatrix}$$

2005

2. (i) $A^2 = \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 1 \times 1 + x \times -1 & 1 \times x + x \times 2 \\ -1 \times 1 + 2 \times -1 & -1 \times x + 2 \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1-x & x+2x \\ -1-2 & -x+4 \end{pmatrix}$$

$$= \begin{pmatrix} 1-x & 3x \\ -3 & -x+4 \end{pmatrix}$$

(ii) deter of $A^2 = 9$

$$9 = (1-x) \times (-x+4) - (-3 \times 3x)$$

$$9 = -x + 4 + x^2 - 4x + 9x$$

$$9 - 4 = -x - 4x + 9x + x^2$$

$$5 = 4x + x^2$$

$$x^2 + 4x - 5 = 0$$

$$x^2 - x + 5x - 5 = 0$$

$$x(x-1) + 5(x-1) = 0$$

$$(x+5)(x-1) = 0$$

$$x+5=0 \text{ or } x-1=0$$

$$\mathbf{x = -5 \text{ or } x = 1}$$

$$\text{prod} = -5 \quad \text{sum} = 4 \quad \text{factors} = -1, 5$$

$$\begin{aligned}
 \text{(ii) } AB &= \begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \\
 &= \begin{pmatrix} 2 \times 2 + 3 \times 3 \\ -1 \times 2 + 5 \times 3 \end{pmatrix} \\
 &= \begin{pmatrix} 13 \\ 13 \end{pmatrix}
 \end{aligned}$$

2013

$$\begin{aligned}
 10. \text{(i) } 2P &= 2 \begin{pmatrix} -1 & -1 \\ 4 & 2 \\ 2 & 0 \end{pmatrix} \\
 2P &= \begin{pmatrix} -2 & -2 \\ 8 & 4 \\ 4 & 0 \end{pmatrix}
 \end{aligned}$$

$$\text{(ii) } \det Q = \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}$$

$$\det Q = (2 \times 1) - (4 \times -1)$$

$$\det Q = 2 + 4$$

$$\det Q = 6$$

$$\text{(iii) } AP = \begin{pmatrix} 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 4 & 2 \\ 2 & 0 \end{pmatrix}$$

$$= (2 \times -1 + 2 \times 4 + 2 \times -1 \quad 2 \times -1 + 2 \times 2 + -1 \times 0)$$

$$= (-2 + 4 - 2 \quad -2 + 4 - 0)$$

$$= (4 \quad 2)$$

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2014

$$11. \text{(i) } A^{-1}$$

$$\det A = \begin{vmatrix} 5 & 2 \\ 1 & 0 \end{vmatrix}$$

$$\det A = (5 \times 0) - (1 \times 2)$$

$$\det A = 0 - 2$$

$$\det A = -2$$

$$A^{-1} = -\frac{1}{2} \begin{pmatrix} 0 & -2 \\ -1 & 4 \end{pmatrix}$$

$$\begin{aligned}
 \text{(ii) } 3A - B &= 3 \begin{pmatrix} 5 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 15 & 6 \\ 3 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 15 + 1 & 6 - 0 \\ 3 - 0 & 0 + 1 \end{pmatrix} \\
 &= \begin{pmatrix} 16 & 6 \\ 3 & 1 \end{pmatrix}
 \end{aligned}$$

$$15.(i) \text{deter } M = (3 \times x) - (5 \times -2)$$

$$22 = 3x - (-10)$$

$$22 = 3x + 10$$

$$22 - 10 = 3x$$

$$\frac{12}{3} = \frac{3x}{3}$$

$$\mathbf{x = 4}$$

$$(ii) M^{-1} = \frac{1}{\text{Deter } M} \begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix}$$

$$\mathbf{M^{-1} = \frac{1}{22} \begin{pmatrix} 4 & 2 \\ -5 & 3 \end{pmatrix}}$$

2018 GCE

$$16.(i) \text{deter } A = (2x \times x) - (2 \times 3)$$

$$12 = 2x^2 - 6$$

$$12 + 6 = 2x^2$$

$$\frac{18}{2} = \frac{2x^2}{2}$$

$$x^2 = 9$$

$$\sqrt{x^2} = \pm\sqrt{9}$$

$$x = \pm 3$$

$$\therefore x = 3$$

$$(ii) A^{-1}$$

$$A = \begin{pmatrix} 2 \times 3 & 2 \\ 3 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 \\ 3 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{12} \begin{pmatrix} 3 & -2 \\ -3 & 6 \end{pmatrix}$$

$$17.(i) \text{deter } A = (4 \times 2) - (1 \times -5)$$

$$\text{Deter } A = 8 - (-5)$$

$$\text{Deter } A = 8 + 5$$

$$\mathbf{\text{Deter } A = 13}$$

$$(ii) B^{-1}$$

$$\therefore \mathbf{B^{-1} = \frac{1}{13} \begin{pmatrix} 5 & -9 \\ -3 & 8 \end{pmatrix}}$$

$$\text{Deter } B = (8 \times 5) - (3 \times y)$$

$$13 = 40 - 3y$$

$$3y = 40 - 13$$

$$\frac{3y}{3} = \frac{27}{3}$$

$$\mathbf{y = 9}$$

(b) Shortest distance

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times 163.35 \times h$$

$$\frac{3064.18}{81.675} = \frac{81.675h}{81.675}$$

$$h = 37.51674319$$

S.D = 37.52km

$$(c) h^2 = m^2 + n^2$$

$$(80)^2 = m^2 + (37.52)^2$$

$$6400 = m^2 + 1407.7504$$

$$m^2 = 6400 - 1407.7504$$

$$\sqrt{m^2} = \sqrt{4992.2496}$$

$$m = 70.65585326$$

HN = 70.65km**2010**

$$4. (a) (i) (PR)^2 = p^2 + r^2 - 2pr \cos \theta$$

$$(PR)^2 = (13.2)^2 + (7.6)^2 - 2(13.2)(7.6)\cos 120^\circ$$

$$(PR)^2 = 174.24 + 57.76 - 200.64\cos 120^\circ$$

$$(PR)^2 = 232 + 100.32$$

$$\sqrt{(PR)^2} = \sqrt{332.32}$$

$$PR = 18.22964618$$

PR = 18.2km

$$(ii) A = \frac{1}{2}pr \sin \theta$$

$$A = \frac{1}{2} \times 13.2 \times 7.6 \times \sin 120^\circ$$

$$A = \frac{86.87966851}{2}$$

$$A = 43.43983425$$

A = 43.4km²

(b) Shortest distance

$$S.D = \frac{2 \times A}{base}$$

$$S.D = \frac{2 \times 43.4}{18.2}$$

$$S.D = 4.769230769$$

S.D = 4.8km