The question is: what is the combination of a set? The question mentions: Historically, the firm has averaged three bats sold for each glove sold. In light of this information, we can understand that each set consists of three bats and one glove. We have to convert all information for each set.

Requirement - 1: 3 bats and 1 glove = 1 set

Contribution per set = [($10 as selling price per bat – $6 as variable cost per bat) × 3 bats per set] + [($15 as selling price per glove – $10 as variable cost per glove) × 1 glove per set) = $17

Breakeven sales quantity = Fixed cost ÷ Contribution per set = $170,000 ÷ $17 = 10,000 sets

Therefore: Number of bats to sold for breakeven = 10,000 sets × 3 bats per set = 30,000 bats

Number of gloves to be sold for breakeven = 10,000 sets × 1 glove per set = 10,000 gloves

Breakeven sales amount = [(3 bats per set × $10 as selling price per bat) + (1 glove per set × $15 per glove)] × 10,000 sets as breakeven sales quantity = $450,000

If even more than two products are given in the question, you can calculate BEP in the same way.

Requirement - 2: Expected sales quantity = (Fixed cost + Expected profit) ÷ Contribution per set

= ($170,000 as fixed cost + $85,000 as expected profit) ÷ $17 as contribution per set = 15,000 sets

Number of bats to sold = 15,000 sets × 3 bats per set = 45,000 bats

Number of gloves to be sold = 15,000 sets × 1 glove per set = 15,000 gloves

Expected sales amount = [(3 bats per set × $10 as selling price per bat) + (1 glove per set × $15 per glove)] × 15,000 sets as breakeven sales quantity = $675,000

So we see expected profit is pre-tax profit. But requirement # 3 says after-tax profit. Hence first we have to turn the after-tax profit into pre-tax profit. The next step is as same as requirement # 2.

Requirement - 3: After-tax profit = Pre-tax profit × (100% – Tax rate)

Pre-tax profit = After-tax profit ÷ (100% – Tax rate) = $85,000 ÷ (100% – 20%) = $106,250

Expected sales quantity = (Fixed cost + Expected profit) ÷ Contribution per set

= ($170,000 as fixed cost + $106,250 as expected profit) ÷ $17 as contribution per set = 16,250 sets

Number of bats to sold = 16,250 sets × 3 bats per set = 48,750 bats

Number of gloves to be sold = 16,250 sets × 1 glove per set = 16,250 gloves

Expected sales amount = [(3 bats per set × $10 as selling price per bat) + (1 glove per set × $15 per glove)] × 16,250 sets as breakeven sales quantity = $731,250

If there is no limiting factor, what would be the optimal production mix? Yes, the maximum demand.

And if there is more than one limiting factor, then the optimal production mix is determined by using linear equation. That is the topic of operations management.

We told earlier that, if the question mentions a minimum production requirement for one or more products, this will get priority over the optimal product mix. Follow the illustration below.

Illustration # 34: Redo Illustration # 33, assuming that 500 units of each of products X and Y are to be produced under a contracted supply. Maximum demand includes / does not include this quantity.

Solution: Steps # 1, 2 and 3 - same as above

Maximum demand *includes* the minimum production requirement:

Step # 4 - Determining the optimal product mix:

First we have to fulfill the minimum production requirement under the contracted supply. Next, with the remaining quantity of labor hours, we should produce Y before producing X.

Labor hours available	= 8,000 hours
(–) Labor hour required for minimum production quantity:	
X: (500 units × 2 hours per unit)	= 1,000 hours
Y: (500 units × 1 hour per unit)	= 500 hours
= Remaining quantity of labor hours	= 6,500 hours
(–) Labor hour required for producing remaining quantity of Y	= 4,500 hours (1 hour × 4,500 units)
= Remaining quantity of labor hours	= 2,000 hours
(÷) Labor hour requirement per unit of X	= 2 hours
= Quantity of X can be produced	= 1,000 units

∴ Optimal production mix: X = 1,500 units (partial quantity) and Y = 5,000 units (full quantity).

A meeting of the board of directors has been convened to discuss the budget and to resolve the problem as to the quantity of each product which should be made and sold. A recent market survey reveals following demand: A = 24,000 units; B = 15,000 units; C = 60,000 units.

The sales director proposes that since C yields the lowest contribution margin, it should no longer be produced. After thorough discussion the board decided that a minimum of 10,000 units of each product should be produced. The remaining production capacity would then be allocated so as to achieve the maximum profit possible.

Prepare a statement which shows the maximum profit which could be achieved in the year.

Answer: A = 22,000 units (partial quantity); C = 60,000 units (full quantity);
B = 10,000 units (minimum production quantity); Maximum profit = $900,000.

We have discovered the mechanism of maximizing profit under constrained situations. What about if the constraint had been removed or at least reduced? Yes, production and profit would increase. This may be possible sometimes, but often at higher cost. That means the additional quantity may not be sourced at the regular price of the restricted or available quantity. For example: when raw material is not available locally for meeting entire demand, we may buy additional raw materials from abroad. In that case the imported price will definitely be higher than domestic price. In the same way, we may increase labor hour availability up to a certain limit by deploying the workers for extra hours. But the overtime hourly pay rate is often much higher than the normal hourly pay rate. Thus the available quantity of a limiting factor may be increased to a certain extent at a higher cost. Now the question is: **how much extra price we should pay for this additional quantity?**

Let us find the answer to this question through analyzing illustration # 40. The general formula is:

Extra price allowable for each additional unit of the restricted resource = Contribution per unit of the finished product ÷ Quantity of the restricted resource required per unit of the finished product

In illustration # 40, we have curtailed the production of A and B due to the shortage of labor hours.

Curtail in the production of A = 24,000 units as maximum demand – 22,000 units as optimized production quantity = 2,000 units

Curtail in the production of B = 15,000 units as maximum demand – 10,000 units as optimized or minimum production quantity = 5,000 units

Now if extra labor hour is available, we must use it for producing A before producing B, because A has higher contribution per labor hour than B.

Extra labor hour required for producing full quantity of A = 2,000 units as curtail in the production of A × 4 labor hours required per unit of A = 8,000 hours

Extra labor hour required for producing full quantity of B = 5,000 units as curtail in the production of B × 8 labor hours required per unit of B = 40,000 hours

The conclusions are as follows:

- We need minimum 4 additional labor hours for increasing production of A and only in that case, extra hourly rate will be worth paying. In fact, we need additional labor of 4 hours or its multiples for increasing production of A. For <4 additional labor hours available, we should not pay anything (not even the regular rate of $6, because no extra unit of A can be produced).
- We should produce B only if labor hour availability can be increased up to 8,008 hours at least. This is because 8,000 additional labor hours are required for producing full quantity of A and 8 additional labor hours are required for producing any extra unit of B. Once again, if extra labor hour is available more than 8,000 hours but less than 8,008 hours, we should not pay anything.

Keeping these two points in mind, we will find out the extra hourly rate payable.

If only 4 additional labor hours are available, we can produce 1 additional unit of A. Then:

Maximum profit = (22,001 units of A × $40 as contribution margin per unit of A) + (10,000 units of B × $60 as contribution margin per unit of B) + (60,000 units of C × $12 as contribution margin per unit of C) - $1,300,000 as fixed cost = $900,040

Increase in maximum profit = $900,040 – $900,000 = $40 = Contribution margin per unit of A

Highest hourly premium (i.e. extra cost) allowable per additional labor hour = Contribution per unit of the finished product ÷ Quantity of the restricted resource required per unit of the finished product = $40 as contribution per unit of A ÷ 4 labor hours required per unit of A = $10

Highest hourly rate allowable for extra labor hours = $6 as regular rate + $10 as premium = $16

That means, for producing the 22001st unit of A, we are ready to pay maximum @ $16 per labor hour, before incurring any loss. If we pay lesser, we can make additional profit.

Please note, for producing the Optimal production quantity (i.e. A = 22,000 units, B = 10,000 units and C = 60,000 units), labor hour rate is $6. The higher rate of $16 applies for extra production. In other words, labor hour rate is $6 for up to 228,000 hours and $16 maximum for next 8,000 hours.

So if (228,000 + 8,000) = 236,000 labor hours are available, we can produce full quantity of A and C and minimum quantity of B. What about if more than 236,000 labor hours are available?

Definitely we will increase the production of B beyond the minimum quantity. But, for this, as we said earlier, minimum 236,008 labor hours are required. If, say 236,006 labor hours are available, we cannot increase the production of B.

Now if 236,008 labor hours are available, then what is the highest extra hourly rate we should pay?

Highest hourly premium (i.e. extra cost) allowable per additional labor hour = Contribution per unit of the finished product ÷ Quantity of the restricted resource required per unit of the finished product = $60 as contribution per unit of B ÷ 8 labor hours required per unit of B = $7.50

Highest hourly rate allowable for extra labor hours = $6 as regular rate + $7.50 as premium = $13.50

So labor hour rate is $6 for up to 228,000 hours and $16 maximum for next 8,000 hours and $13.50 maximum for further 40,000 hours. After that, entire demand is fulfilled.

Please note carefully that fixed cost cannot increase due to the increase in production and sales.

Hourly rate actually *payable* > Highest hourly rate *allowable*

So Survey Ltd. should not increase production of B, and only 8,000 extra labor hours should be used.

Optimal production mix: A = 24,000 units (full quantity);

B = 10,000 units (minimum production requirement); C = 60,000 units (full quantity).

Step # 2 - Calculating the maximum profit:

	Maximum profit under original situation	=	$900,000
(+)	Contribution from additional quantity of A (2,000 units × $40)	=	$80,000
(–)	Overtime premium [8,000 hours × ($15 – $6)]	=	($72,000)
=	Maximum profit under original situation	=	$908,000

Illustration # 42: In Illustration # 40, assume that additional labor hours are available as follows:

Additional labor hour	Hourly premium
1 hour - 5,000 hours	$5
5,001 hours - 10,000 hours	$6
10,001 hours - 12,000 hours	$7
12,001 hours - 15,000 hours	$8

Find out the optimal production mix and the maximum profit under the changed situation.

Solution: Step # 1 - Calculating the optimal production mix:

Extra labor hours required for producing full quantity of A = 8,000 hours
Extra labor hours required for producing full quantity of B = 40,000 hours
Total extra labor hours required = 48,000 hours Extra labor hours available = 12,000 hours

So Survey Ltd. can produce full quantity of A and some additional quantity of B, if the increased production is worth paying the hourly premium.

In case of A, highest hourly premium allowable for each extra labor hour = $10
In case of B, highest hourly premium allowable for each extra labor hour = $7.50

So far we have dealt with all final products. Sometimes the question may ask you to deal with the inputs instead (e.g. raw materials, labor etc.). In such a case, the question will mention in-house production cost of the inputs and purchase price of the same from local or outside source, or subcontracting price. Hence the production preference will be defined in a different manner.

If the question mentions:		Production preference is defined on the basis of:
Selling price of the final product	→	Contribution per unit of the constrained resource
Purchase price of inputs from local source	→	Variable cost saved from in-house production per unit of limiting factor

For your information, this type of analysis is known as **make or buy decision.**

Illustration # 45: Please refer to the following information:

	Standard	Premium
Variable cost per unit	$2	$4
Raw materials consumption per unit	2 kg	2.5 kg
Intended production	10,000 units	8,000 units
Purchase price per unit from local sources	$7	$8

Assuming 32,000 kg of raw materials are available during the period, determine the optimal plan. Also calculate total variable cost to be incurred for the period.

Solution: Raw materials requirement = (10,000 units of Standard product × 2 kg per unit) + (8,000 units of Premium product × 2.5 kg per unit) = 40,000 kg

Raw materials availability = 32,000 kg. So raw materials is the limiting factor.

		Standard	Premium
	Variable cost per unit saved from making	($7 – $2) = $5	($8 – $4) = $4
(÷)	Raw materials requirement	2 kg	2.5 kg
=	Variable cost saved per kg of raw materials	$2.5	1.6

So production preference is first the Standard product and then the Premium product.

Raw materials required for producing full quantity of the Standard product = 20,000 kg

Quantity of the Premium product can be produced by the remaining raw materials = 4,800 units

Quantity of the Premium product to be procured from local sources = 3,200 units

Total variable cost to be incurred for the period = (10,000 units of the Standard product × $2 as the variable cost per unit of in-house production) + (4,800 units of the Premium product × $4 as variable cost per unit of in-house production) + (3,200 units of the Premium product × $8 as purchase price per unit from local sources) = $64,800