$$\frac{d}{d\theta}(\cot\theta) = -\operatorname{cosec}^2\theta$$

$$d(\cot\theta) = -\operatorname{cosec}^2\theta\, d\theta \quad ------------ \quad (7)$$

For some constant K

$$d(\cot\theta) = d(\cot\theta) + 0 = d(\cot\theta) + d(K) = d(\cot\theta + K) \quad --------- \quad (8)$$

From equation (7) & *equation* (8)

$$-\operatorname{cosec}^2\theta\, d\theta = d(\cot\theta + K)$$

Taking integrals on both sides

$$\int -\operatorname{cosec}^2\theta\, d\theta = \int d(\cot\theta + K) = \cot\theta + K$$

$$-\int \operatorname{cosec}^2\theta\, d\theta = \cot\theta + k$$

$$\int \operatorname{cosec}^2\theta\, d\theta = -\{\cot\theta + K\} = -\cot\theta - K$$

Putting $-K = C$

$$\int \operatorname{cosec}^2\theta\, d\theta = -\cot\theta + C$$

....

$$y = \frac{1}{\sin\theta}$$

Using the formula

$$\frac{d}{d\theta}\left(\frac{u}{v}\right) = \frac{v\frac{du}{d\theta} - u\frac{dv}{d\theta}}{v^2}$$

Here

$u = 1, v = \sin\theta$