Bayes' theorem is given by:

$P(A|B) = P(B|A) * P(A) / P(B)$

where P(A|B) is the conditional probability of A given B, P(B|A) is the conditional probability of B given A, P(A) is the prior probability of A, and P(B) is the prior probability of B.

Independent Events

Two events A and B are said to be independent if the occurrence of one does not affect the occurrence of the other. In other words, P(A|B) = P(A) and P(B|A) = P(B). If two events are independent, then the joint probability of both events occurring is the product of their individual probabilities. That is, P(A and B) = P(A) * P(B).

Law of Total Probability

The law of total probability is a theorem that states that the probability of an event A can be calculated as the sum of the probabilities of A given each possible outcome of a second event B, weighted by the probability of each outcome of B. Mathematically, the law of total probability is given by:

$P(A) = Sum(P(A|B) * P(B))$

where B represents all possible outcomes of a second event.

Expectation and Variance of a Random Variable

The expected value or mean of a random variable X is a measure of the central tendency of its probability distribution. It is defined as:

$E[X] = Sum(x * P(X = x))$

The variance of a random variable X is a measure of its spread or variability around its mean. It is defined as: