$\lim \{x - a\} f(x) = L$ 

For example, consider the function  $f(x) = (x^2 - 1)/(x - 1)$ . As x approaches 1, the denominator (x - 1)approaches 0, which would make the function undefined. However, if we simplify the function by factoring the numerator, we get:

f(x) = (x + 1)(x - 1)/(x - 1) = x + 1

Now, as x approaches 1, f(x) approaches 2, which is the limit of the function at x = 1.

### One-sided and Two-sided Limits

When we talk about the limit of a function at a point, we can consider both the left-hand and right-hand limits. The left-hand limit is the value that the function approaches as x approaches a from the left (i.e., as x gets smaller and smaller than a), while the right-hand limit is the value that the function approaches as x approaches a from the right (i.e., as x gets larger and larger than a).

If the left-hand and right-hand limits are equal, then we say that the function has a limit at that point. If the left-hand and right-hand limits are not equal, or if one or both of the limits do not exist, then we say that the function does not have a limit at that point.

For example, consider the function f(x) = |x|. As x approaches 0 from the left, f(x) approaches 0, while as x approaches 0 from the right, f(x) approaches 0 as well. Therefore, the limit of f(x) as x approaches 0 exists and is equal to 0.

On the other hand, consider the function q(x) = 1/x. As x approaches 0 from the left, q(x) approaches negative infinity, while as x approaches 0 from the right, g(x) approaches positive infinity. The fore, the

Limits at Infinity In addition to limits at specific points, we can also poss der times as x approaches infinity or negative infinity. If a function f(x) approaches a value to s x becomes very larte (either positively or negatively), we write:

# lim {x->infinity}

mple, consider the function h(x) = 1/x. As x becomes very large (either positively or negatively), For exa h(x) approaches 0. Therefore, the limit of h(x) as x approaches infinity (or negative infinity) is 0. Overall, understanding the concept of a limit is essential for understanding calculus and the behavior of functions. By understanding how a function approaches a value, the different types of limits, and limits at infinity, we can better understand the continuity of functions and their behavior around specific points.

# **3.** Continuity and Limits

- Definition of continuity
- Relationship between limits and continuity
- Identifying discontinuities in functions

In calculus, the concept of limits is fundamental to understanding continuity. A limit is the value that a function approaches as its input approaches a certain value. Continuity refers to the property of a function where the limit of the function at a certain point is equal to the value of the function at that point, we will explore the definition of continuity, the relationship between limits and continuity, and how to identify discontinuities in functions.

## **Definition of Continuity**

A function is said to be continuous at a point if the limit of the function at that point exists and is equal to the value of the function at that point. In other words, a function is continuous if it can be drawn without lifting the pen from the paper.