EXAMPLE 5.2

Derive the formula for $(F/P)^i_n$.

Solution. For $n = 1$,

$$F = (1 + i)\,P$$

that is,

$$(F/P)^i_1 = (1 + i)^1$$

For any n,

$$F = (1 + i)\,(F/P)^i_{n-1}\,P$$

that is,

$$(F/P)^i_n = (1 + i)\,(F/P)^i_{n-1}$$

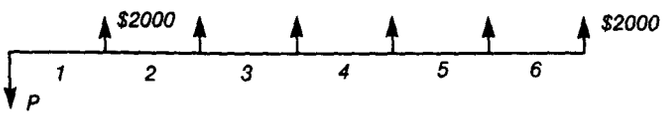
By induction,

$$(F/P)^i_n = (1 + i)^n$$

EXAMPLE 5.3

A new widget twister, with a life of six years, would save $2,000 in production costs each year. Using a 12% interest rate, determine the highest price that could be justified for the machine. Although the savings occur continuously throughout each year, follow the usual practice of lumping all amounts at the ends of years.

Solution. First, sketch the cash flow diagram.



The cash flow diagram indicates that an amount in a P-pattern must be found that is equivalent to $2,000 in an A-pattern. The corresponding equation is

$$P = (P/A)^i_n A$$

$$= (P/A)^{12\%}_6\ 2000$$

Table 5.2 is used to evaluate the interest factor for $i = 12\%$ and $n = 6$:

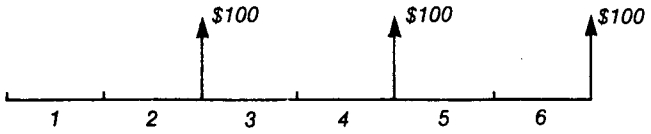
$$P = 4.1114 \times 2000$$

$$= \$8223$$

EXAMPLE 5.13

ANNUAL COMPOUNDING BUT PAYMENT EVERY m YEARS

With interest at 10% compute the present equivalent of



Solution. First convert each payment to an A-pattern for the m preceding years. That is,

$$A = (A/F)^{10\%}_{2}\ 100$$

$$= \$47.62$$

Then, convert the A-pattern to a P-pattern:

$$P = (P/A)^{10\%}_{6}\ 47.62$$

$$= \$207$$

5.6 Evaluating Alternatives

The techniques of engineering economy assume the objective of maximizing net value. For a business, "value" means after-tax cash flow. For a not-for-profit organization, such as a government agency, value may include non-cash benefits, such as, clean air, improved public health, recreation, to which dollar amounts have been assigned.

Sections 5.7 through 5.17 concern strategies for selecting alternatives such that net value is maximized. The logic of these methods will be clear if the following distinctions are made between two different types of interest rates, and between two different types of relationships among alternatives.

TYPES OF INTEREST RATES

Rate of Return (ROR): The estimated interest rate produced by an investment. It may be computed by finding the interest rate such that the estimated income and non-cash benefits (positive value), and the estimated expenditures and non-cash costs (negative value) sum to a net equivalent value of zero.

Minimum Attractive Rate of Return (MARR): The lowest rate of return that the organization will accept. In engineering economy problems, it is usually a given quantity and may be called, somewhat imprecisely, "interest," "interest rate," "cost of money," or "interest on capital."

Inflation, if it is anticipated, can be exploited by fixing costs and allowing income to increase. A manufacturing business can fix its costs by entering long-term contracts for materials and wages, by purchasing materials long before they are needed, and by stockpiling its product for sale later. Income is allowed to respond to inflation by avoiding long-term contracts for the product. Borrowing becomes more attractive if inflation is expected since the debt will be paid with the less valuable cash of the future.

EXAMPLE 5.23

A machine having a five-year life can replace a worker now earning $10,000 per year who is subject to 5% annual "cost of living" increases. Operating and maintenance costs for the machine are negligible. *MARR* is 10%. Find the maximum price that can be justified for the machine if:

a) general price inflation is 5%, and

b) general price inflation is zero.

Solution.

a) Although the worker gets a larger amount of money each year, his raises are exactly matched by increased prices, including those of his employer's product. "Buying power" of his annual wage remains equal to the current value of $10,000. Hence, the maximum justifiable price for the machine is

$$P = (P/A)^{10\%}_{5}\ 10{,}000 = \$37{,}908$$

b) The maximum justifiable price of the machine is equal to the present equivalent value of the annual amounts of the wage:

$$(P/F)^{10\%}_{1}(1.05)\ 10{,}000 = \$\ 9{,}545$$

$$(P/F)^{10\%}_{2}(1.05)^2\ 10{,}000 = \$\ 9{,}112$$

$$(P/F)^{10\%}_{3}(1.05)^3\ 10{,}000 = \$\ 8{,}697$$

$$(P/F)^{10\%}_{4}(1.05)^4\ 10{,}000 = \$\ 8{,}302$$

$$(P/F)^{10\%}_{5}(1.05)^5\ 10{,}000 = \$\ 7{,}925$$

$$\therefore P = \$43{,}581$$

EXAMPLE 5.24

Recompute the value, in terms of 1987 "buying power," of the "Acme 8% of 2000" bond discussed in Ex. 5.5, but assume 6% annual inflation.

Solution. The cash flow for each year must be divided by an inflation factor as well as multiplied by an

TABLE 5.2. Compound Interest Factors (continued)

$i = 1.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9901	0.9901	-0.0000	1.0100	1.0000	1.0100	1.0000	-0.0000	1
2	.9803	1.9704	0.9803	1.0201	2.0100	0.5075	0.4975	0.4975	2
3	.9706	2.9410	2.9215	1.0303	3.0301	0.3400	0.3300	0.9934	3
4	.9610	3.9020	5.8044	1.0406	4.0604	0.2563	0.2463	1.4876	4
5	.9515	4.8534	9.6103	1.0510	5.1010	0.2060	0.1960	1.9801	5
6	.9420	5.7955	14.3205	1.0615	6.1520	0.1725	0.1625	2.4710	6
7	.9327	6.7282	19.9168	1.0721	7.2135	0.1486	0.1386	2.9602	7
8	.9235	7.6517	26.3812	1.0829	8.2857	0.1307	0.1207	3.4478	8
9	.9143	8.5660	33.6959	1.0937	9.3685	0.1167	0.1067	3.9337	9
10	.9053	9.4713	41.8435	1.1046	10.4622	0.1056	0.0956	4.4179	10
11	.8963	10.3676	50.8067	1.1157	11.5668	0.0965	0.0865	4.9005	11
12	.8874	11.2551	60.5687	1.1268	12.6825	0.0888	0.0788	5.3815	12
13	.8787	12.1337	71.1126	1.1381	13.8093	0.0824	0.0724	5.8607	13
14	.8700	13.0037	82.4221	1.1495	14.9474	0.0769	0.0669	6.3384	14
15	.8613	13.8651	94.4810	1.1610	16.0969	0.0721	0.0621	6.8143	15
16	.8528	14.7179	107.2734	1.1726	17.2579	0.0679	0.0579	7.2886	16
17	.8444	15.5623	120.7834	1.1843	18.4304	0.0643	0.0543	7.7613	17
18	.8360	16.3983	134.9957	1.1961	19.6147	0.0610	0.0510	8.2323	18
19	.8277	17.2260	149.8950	1.2081	20.8109	0.0581	0.0481	8.7017	19
20	.8195	18.0456	165.4664	1.2202	22.0190	0.0554	0.0454	9.1694	20
21	.8114	18.8570	181.6950	1.2324	23.2392	0.0530	0.0430	9.6354	21
22	.8034	19.6604	198.5663	1.2447	24.4716	0.0509	0.0409	10.0998	22
23	.7954	20.4558	216.0660	1.2572	25.7163	0.0489	0.0389	10.5626	23
24	.7876	21.2434	234.1800	1.2697	26.9735	0.0471	0.0371	11.0237	24
25	.7798	22.0232	252.8945	1.2824	28.2432	0.0454	0.0354	11.4831	25
26	.7720	22.7952	272.1956	1.2953	29.5256	0.0439	0.0339	11.9409	26
28	.7568	24.3164	[illegible]	1.3213	[illegible]	0.0411	0.0311	12.8516	28
30	.7419	25.8[illegible]	355.0021	1.3478	[illegible]	0.0387	0.0287	13.7557	30
∞	.0000	100.0[illegible]	10 000.0	∞	∞	0.0100	0.0000	100.0000	∞

$i = 2.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9804	0.9804	-0.0000	1.0200	1.0000	1.0200	1.0000	-0.0000	1
2	.9612	1.9416	0.9612	1.0404	2.0200	0.5150	0.4950	0.4950	2
3	.9423	2.8839	2.8458	1.0612	3.0604	0.3468	0.3268	0.9868	3
4	.9238	3.8077	5.6173	1.0824	4.1216	0.2626	0.2426	1.4752	4
5	.9057	4.7135	9.2403	1.1041	5.2040	0.2122	0.1922	1.9604	5
6	.8880	5.6014	13.6801	1.1262	6.3081	0.1785	0.1585	2.4423	6
7	.8706	6.4720	18.9035	1.1487	7.4343	0.1545	0.1345	2.9208	7
8	.8535	7.3255	24.8779	1.1717	8.5830	0.1365	0.1165	3.3961	8
9	.8368	8.1622	31.5720	1.1951	9.7546	0.1225	0.1025	3.8681	9
10	.8203	8.9826	38.9551	1.2190	10.9497	0.1113	0.0913	4.3367	10
11	.8043	9.7868	46.9977	1.2434	12.1687	0.1022	0.0822	4.8021	11
12	.7885	10.5753	55.6712	1.2682	13.4121	0.0946	0.0746	5.2642	12
13	.7730	11.3484	64.9475	1.2936	14.6803	0.0881	0.0681	5.7231	13
14	.7579	12.1062	74.7999	1.3195	15.9739	0.0826	0.0626	6.1786	14
15	.7430	12.8493	85.2021	1.3459	17.2934	0.0778	0.0578	6.6309	15
16	.7284	13.5777	96.1288	1.3728	18.6393	0.0737	0.0537	7.0799	16
17	.7142	14.2919	107.5554	1.4002	20.0121	0.0700	0.0500	7.5256	17
18	.7002	14.9920	119.4581	1.4282	21.4123	0.0667	0.0467	7.9681	18
19	.6864	15.6785	131.8139	1.4568	22.8406	0.0638	0.0438	8.4073	19
20	.6730	16.3514	144.6003	1.4859	24.2974	0.0612	0.0412	8.8433	20
21	.6598	17.0112	157.7959	1.5157	25.7833	0.0588	0.0388	9.2760	21
22	.6468	17.6580	171.3795	1.5460	27.2990	0.0566	0.0366	9.7055	22
23	.6342	18.2922	185.3309	1.5769	28.8450	0.0547	0.0347	10.1317	23
24	.6217	18.9139	199.6305	1.6084	30.4219	0.0529	0.0329	10.5547	24
25	.6095	19.5235	214.2592	1.6406	32.0303	0.0512	0.0312	10.9745	25
26	.5976	20.1210	229.1987	1.6734	33.6709	0.0497	0.0297	11.3910	26
28	.5744	21.2813	259.9392	1.7410	37.0512	0.0470	0.0270	12.2145	28
30	.5521	22.3965	291.7164	1.8114	40.5681	0.0446	0.0246	13.0251	30
∞	.0000	50.0000	2500.0000	∞	∞	0.0200	0.0000	50.0000	∞

TABLE 5.2. Compound Interest Factors (continued)

$i = 3\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	0.97087	0.9708	0.0000	1.0300	1.0000	1.03000	1.00000	0.0000	1
2	0.94260	1.9134	0.9426	1.0609	2.0300	0.52261	0.49261	0.4926	2
3	0.91514	2.8286	2.7728	1.0927	3.0909	0.35353	0.32353	0.9803	3
4	0.88849	3.7171	5.4383	1.1255	4.1836	0.26903	0.23903	1.4630	4
5	0.86261	4.5797	8.8887	1.1592	5.3091	0.21835	0.18835	1.9409	5
6	0.83748	5.4171	13.076	1.1940	6.4684	0.18460	0.15460	2.4138	6
7	0.81309	6.2302	17.954	1.2298	7.6624	0.16051	0.13051	2.8818	7
8	0.78941	7.0196	23.480	1.2667	8.8923	0.14246	0.11246	3.3449	8
9	0.76642	7.7861	29.611	1.3047	10.159	0.12843	0.09843	3.8031	9
10	0.74409	8.5302	36.308	1.3439	11.463	0.11723	0.08723	4.2565	10
11	0.72242	9.2526	43.533	1.3842	12.807	0.10808	0.07808	4.7049	11
12	0.70138	9.9540	51.248	1.4257	14.192	0.10046	0.07046	5.1485	12
13	0.68095	10.635	59.419	1.4685	15.617	0.09403	0.06403	5.5872	13
14	0.66112	11.296	68.014	1.5125	17.086	0.08853	0.05853	6.0210	14
15	0.64186	11.937	77.000	1.5579	18.598	0.08377	0.05377	6.4500	15
16	0.62317	12.561	86.347	1.6047	20.156	0.07961	0.04961	6.8742	16
17	0.60502	13.166	96.028	1.6528	21.761	0.07595	0.04595	7.2935	17
18	0.58739	13.753	106.01	1.7024	23.414	0.07271	0.04271	7.7081	18
19	0.57029	14.323	116.27	1.7535	25.116	0.06981	0.03981	8.1178	19
20	0.55368	14.877	126.79	1.8061	26.870	0.06722	0.03722	8.5228	20
21	0.53755	15.415	137.55	1.8602	28.676	0.06487	0.03487	8.9230	21
22	0.52189	15.936	148.50	1.9161	30.536	0.06275	0.03275	9.3185	22
23	0.50669	16.443	159.65	1.9735	32.452	0.06081	0.03081	9.7093	23
24	0.49193	16.935	170.97	2.0327	34.426	0.05905	0.02905	10.095	24
25	0.47761	17.413	182.43	2.0937	36.459	0.05743	0.02743	10.476	25
26	0.46369	17.876	194.02	2.1565	38.553	0.05594	0.02594	10.853	26
28	0.43708	18.764	217.53	2.2879	42.93	0.05329	0.02329	11.593	28
30	0.41199	19.600	241.36	2.4272	47.575	0.05102	0.02102	12.314	30
∞	0	33.333	1111.11	∞	∞	0.03000	0.0000	33.333	∞

$i = 4.00\%$

n	(P/F)	(P/A)	(P/G)	(F/P)	(F/A)	(A/P)	(A/F)	(A/G)	n
1	.9615	0.9615	-0.0000	1.0400	1.0000	1.0400	1.0000	-0.0000	1
2	.9246	1.8861	0.9246	1.0816	2.0400	0.5302	0.4902	0.4902	2
3	.8890	2.7751	2.7025	1.1249	3.1216	0.3603	0.3203	0.9739	3
4	.8548	3.6299	5.2670	1.1699	4.2465	0.2755	0.2355	1.4510	4
5	.8219	4.4518	8.5547	1.2167	5.4163	0.2246	0.1846	1.9216	5
6	.7903	5.2421	12.5062	1.2653	6.6330	0.1908	0.1508	2.3857	6
7	.7599	6.0021	17.0657	1.3159	7.8983	0.1666	0.1266	2.8433	7
8	.7307	6.7327	22.1806	1.3686	9.2142	0.1485	0.1085	3.2944	8
9	.7026	7.4353	27.8013	1.4233	10.5828	0.1345	0.0945	3.7391	9
10	.6756	8.1109	33.8814	1.4802	12.0061	0.1233	0.0833	4.1773	10
11	.6496	8.7605	40.3772	1.5395	13.4864	0.1141	0.0741	4.6090	11
12	.6246	9.3851	47.2477	1.6010	15.0258	0.1066	0.0666	5.0343	12
13	.6006	9.9856	54.4546	1.6651	16.6268	0.1001	0.0601	5.4533	13
14	.5775	10.5631	61.9618	1.7317	18.2919	0.0947	0.0547	5.8659	14
15	.5553	11.1184	69.7355	1.8009	20.0236	0.0899	0.0499	6.2721	15
16	.5339	11.6523	77.7441	1.8730	21.8245	0.0858	0.0458	6.6720	16
17	.5134	12.1657	85.9581	1.9479	23.6975	0.0822	0.0422	7.0656	17
18	.4936	12.6593	94.3498	2.0258	25.6454	0.0790	0.0390	7.4530	18
19	.4746	13.1339	102.8933	2.1068	27.6712	0.0761	0.0361	7.8342	19
20	.4564	13.5903	111.5647	2.1911	29.7781	0.0736	0.0336	8.2091	20
21	.4388	14.0292	120.3414	2.2788	31.9692	0.0713	0.0313	8.5779	21
22	.4220	14.4511	129.2024	2.3699	34.2480	0.0692	0.0292	8.9407	22
23	.4057	14.8568	138.1284	2.4647	36.6179	0.0673	0.0273	9.2973	23
24	.3901	15.2470	147.1012	2.5633	39.0826	0.0656	0.0256	9.6479	24
25	.3751	15.6221	156.1040	2.6658	41.6459	0.0640	0.0240	9.9925	25
26	.3607	15.9828	165.1212	2.7725	44.3117	0.0626	0.0226	10.3312	26
28	.3335	16.6631	183.1424	2.9987	49.9676	0.0600	0.0200	10.9909	28
30	.3083	17.2920	201.0618	3.2434	56.0849	0.0578	0.0178	11.6274	30
∞	.0000	25.000	625.0000	∞	∞	0.0400	0.0000	25.0000	∞