

UNITS & MEASUREMENTS

MEASUREMENT OF MASS & TIME

MASS

• Unified atomic mass unit (amu) is used to measure mass of atoms & molecules

• $1\text{amu} = (1/12)^{\text{th}}$ mass of one ^{12}C atom

• $1\text{amu} = 1.66 \times 10^{-27} \text{ kg}$

• Electron mass - 10^{-30} kg

• Earth mass : 10^{25} kg

• Observable Universe 10^{55} kg

TIME

• SI unit is second (based on caesium clock with an uncertainty less than 1 part in 10^{-13} ie, 3µs loss every year)

• Timespan of unstable particle: 10^{-24} s

• Age of universe: 10^{17} s

MEASUREMENT OF LENGTH

• Large distance is measured by parallax method

• Parallax angle = $\frac{\text{BASIS}}{\text{DISTANCE}} = \frac{b}{x}$

• $1^\circ = 1.745 \times 10^{-2} \text{ rad}$

• $1' = 2.91 \times 10^{-4} \text{ rad}$

• $1'' = 4.85 \times 10^{-6} \text{ rad}$

• For very small sizes, optical microscope, tunneling microscope, electron microscope were used.

• $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$

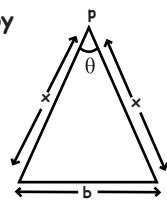
• $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$

• $1 \text{ parsec} = 3.08 \times 10^{16} \text{ m}$

• Size of proton 10^{-15} m

• Radius of Earth: 10^7 m

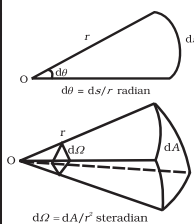
• Distance to Boundary of Observable Universe : 10^{26} m



SI SYSTEM

7 Base units and 2 supplementary units

NO.	Base Units		
	Quantity	Unit	Symbol
1	Length	meter	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol



NO.	Supplementary Units		
	Quantity	Unit	Symbol
1	Plane angle	radian	rad
2	Solid angle	steradian	sr

Q. Unit of permittivity of free space ϵ_0 is

- (a) coulomb/newton-metre
- (b) newton-metre² /coulomb²
- (c) coulomb² /newton-metre²
- (d) coulomb² / (newton-metre)²

SIGNIFICANT FIGURES

The digits in a measured quantity which are reliable and confidence in our measurement + the digit which is uncertain.

RULES FOR SIGNIFICANT FIGURES

1. All non-zero digits are significant. For example, 42.3 has three significant figures; 243.4 has four significant figures; and 24.123 has five significant figures.

2. A zero becomes significant figure if it appears between two non-zero digits. For example, 5.03 has three significant figures; 5.604 has four significant figures; and 4.004 has four significant figures.

3. Leading zeros or the zeros placed to the left of the number are never significant. For example, 0.543 has three significant figures; 0.045 has two significant figures; and 0.006 has one significant figure.

4. Trailing zeros or the zeros placed to the right of the number are significant. For example, 4.330 has four significant figures; 433.00 has five significant figures; and 343.000 has six significant figures.

5. In exponential notation, the numerical portion gives the number of significant figures. For example, 1.32×10^{-2} has three significant figures and 1.32×10^4 has three significant figures.

RULES FOR ROUNDING OF A MEASUREMENT

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged. For example, $x = 7.82$ is rounded off to 7.8 and $x = 3.94$ is rounded off to 3.9.

2. If the digit to be dropped is more than 5, then the preceding digit is raised by one. For example, $x = 6.87$ is rounded off to 6.9 and $x = 12.78$ is rounded off to 12.8.

3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. For example, $x = 16.351$ is rounded off to 16.4 and $x = 6.758$ is rounded off to 6.8.

4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is even, is left unchanged. For example, $x = 3.250$ becomes 3.2 on rounding off and $x = 12.650$ becomes 12.6 on rounding off.

5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is odd, is raised by one. For example, $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

RULES FOR ROUNDING OF A MEASUREMENT

ADDITION & SUBTRACTION

In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places

$$\begin{array}{r} 3.1421 \\ 0.241 \\ +0.09 \\ \hline 3.4731 \end{array}$$

← (has two decimal places)
← (Answer should be reported to two decimal places after rounding off)

Answer = 3.47

MULTIPLICATION & DIVISION

When numbers are multiplied or divided, the number of significant figures in the answer equals the smallest number of significant figures in any of the original numbers

$$\begin{array}{r} 51.028 \\ \times 1.31 \\ \hline 66.84668 \end{array}$$

← (Three significant figures)
← (Answer should have three significant figures after rounding off)

Answer = 66.8

If $L = 2.331 \text{ cm}$, $B = 2.1 \text{ cm}$, then $L+B = ?$

- (a) 4.431 cm
- (b) 4.43 cm
- (c) 4.4 cm
- (d) 4 cm

Dimensional Analysis

Dimensions of a physical quantity are power to which units of base quantity are raised. Eg: $[M]^a [L]^b [T]^c [A]^d [K]^e$

APPLICATIONS

checking the correctness of various formulae

Eg: If $Z = A+B$, $[Z] = [A] = [B]$

conversion of one system of unit into another

$n_1 u_1 = n_2 u_2$
Eg: $n_1 [M_1^A L_1^B T_1^C] = n_2 [M_2^A L_2^B T_2^C]$

Deducing relation among physical quantity

$n_1 = n_2 \left[\frac{M_2}{M_1} \right]^A \left[\frac{L_2}{L_1} \right]^B \left[\frac{T_2}{T_1} \right]^C$

DIMENSIONAL FORMULA

- 1) Pressure = stress = Young's modulus = $ML^{-1} T^{-2}$
- 2) Work = Energy = Torque = $M L^2 T^{-2}$
- 3) Power $P = M L^2 T^{-3}$
- 4) Gravitational constant $G = M^{-1} L^3 T^{-2}$
- 5) Force constant = Spring constant = $M T^{-2}$
- 6) Coefficient of viscosity = $M L^{-1} T^{-1}$
- 7) Latent heat $L = L^2 T^{-2}$
- 8) Electric potential = $\frac{P}{I} = M L^2 T^{-3} A^{-1}$
- 9) Resistance = $\sqrt{\frac{\mu_0}{\epsilon_0}} = M L^2 T^{-3} A^{-2}$
- 10) Capacitance = $M^{-1} L^{-2} T^{-4} A^2$
- 11) Permittivity $\epsilon_0 = M^{-1} L^{-3} T^4 A^2$
- 12) Angular momentum = planck's constant = $M L^2 T^{-1}$

$$\begin{aligned} T &\propto \sqrt{\frac{l}{g}} \propto \sqrt{\frac{m}{k}} \propto \sqrt{\frac{R}{g}} \\ \text{Time period } \frac{L}{R} &= RC = \sqrt{LC} \end{aligned}$$

DIMENSIONLESS QUANTITIES

- 1) Strain
- 2) Refractive index
- 3) Relative density
- 4) Plane angle
- 5) Solid angle

In SI Units, the dimensions of $\sqrt{\frac{\epsilon_0}{\mu_0}}$ is:

- a) $A^{-1} T M L^3$
- b) $A T^2 M^{-1} L^{-1}$
- c) $A T^{-3} M L^{3/2}$
- d) $A^2 T^3 M^{-1} L^{-2}$

INSTRUMENTS

Least Count: Smallest quantity an instrument can measure

mm scale 1mm
vernier scale 0.1mm
screw gauge 0.01mm

VERNIER CALIPERS

Least Count = 1 MSD - 1 VSD

If n VSD Coincides with (n-1) MSD, then (n-1) MSD = n VSD
 $\therefore 1 \text{ VSD} = \frac{n-1}{n} \text{ MSD}$

Least Count = 1 MSD - $\frac{n-1}{n} \text{ MSD} = \frac{1 \text{ MSD}}{n}$

Total Reading = Main Scale Reading + coinciding Vernier Scale division $\times$ least count

In a vernier calipers, one main scale division is x cm & n division of vernier scale coincide with n-1 divisions of the main scale. the least count (in cm) of the calipers is.

- a) $\left(\frac{n-1}{n}\right) \times$
- b) $\frac{nx}{(n-1)}$
- c) $\frac{x}{(n-1)}$
- d) $\frac{x}{n}$

SCREW GAUGE

Pitch = $\frac{\text{Main Scale Reading}}{\text{No. of rotations}}$

Least Count = $\frac{\text{pitch}}{\text{Total no. of divisions on circular scale}}$

Total Reading = Linear Scale Reading + circular scale reading $\times$ least count

The least count of the main scale of a screw gauge is 1mm the minimum no. of divisions on its circular scale required to measure 5µm diameter of wire is

- a) 200
- b) 50
- c) 400
- d) 100

ERRORS IN MEASUREMENT

Difference between true value & measured value of a quantity

Systematic Errors
Errors which tend to occur only in one direction, either positive or negative

Random Errors
Irregular and at random in magnitude & direction

Instrumental
Due to inbuilt defect of measuring instrument

Experimental
Limitations in experimental technique

Personal
Due to individual bias, Lack of proper setting of apparatus

• Least count error is the smallest value that can be measured by instrument (occurs with random & systematic errors)

• Absolute Error :- $\Delta a = |a - a_{\text{mean}}|$, $a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$

• Relative Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$; $\Delta a_{\text{mean}} = \frac{\Delta a_1 + \Delta a_2 + \Delta a_3 + \dots + \Delta a_n}{n}$

• Percentage Error :- $\frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100$

COMBINATION OF ERRORS

Operations	Formula Z	Absolute error ΔZ	Relative error $\Delta Z/Z$	Percentage error $100 \times \Delta Z/Z$
Sum	$A+B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A+B}$	$\frac{\Delta A + \Delta B}{A+B} \times 100$
Difference	$A-B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A-B}$	$\frac{\Delta A + \Delta B}{A-B} \times 100$
Multiplication	$A \times B$	$A \Delta B + B \Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) \times 100$
Division	$\frac{A}{B}$	$\frac{B \Delta A + A \Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B} \right) \times 100$
Power	A^n	$n A^{n-1} \Delta A$	$n \frac{\Delta A}{A}$	$n \frac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\frac{1}{n} A^{1/n-1} \Delta A$	$\frac{1}{n} \frac{\Delta A}{A}$	$\frac{1}{n} \frac{\Delta A}{A} \times 100$

General rule:

If $Z = \frac{A^p B^q}{C^r}$, Then the maximum fractional relative error in Z will be:

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

In an experiment four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4% respectively. Quantity P is calculated as follows:

$$P = \frac{a^2 b^2}{cd} \text{ then percentage error in P is}$$

- (a) 14%
- (b) 10%
- (c) 7%
- (d) 4%

