



Learn about It!

In the figures shown in *Warm Up!*, for you to arrive with the next figure, you tried to observe the first four figures and looked for a pattern. Doing this, you have somehow applied what is called inductive reasoning.

To understand this type of reasoning, consider the following problem.



James leaves for school at 7:00 in the morning. He is always on time for his classes. James assumes then that he will always be on time if he leaves at 7:00 a.m.

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Definition 1.1: Logic is the science of correct thinking.

Definition 1.2: Reasoning is the process of drawing conclusions or inferences from facts or premises.

There are two kinds of reasoning: *inductive* and *deductive*.

Definition 1.3: Inductive Reasoning is a kind of reasoning based on repeated observations and is characterized by drawing a conclusion called *conjecture* from observations of specific examples.

Try It Yourself!

A school's mountaineering club started with 8 members on the first week. Then it had 16 members on the second week, 32 on the third, and 64 on the fourth week. How many members did it have on the fifth week?

**Check Your Understanding!**

1. Determine whether each of the following arguments is a result of induction.
 - a. Katarina's family has a new family neighbor. The eldest of that family has his name that starts with X, so does the second. Katarina supposes that the name of the third also starts with X.
 - b. On Thursdays, we wear green. Today is Thursday, so we shall wear green.
 - c. Glenda passed her first math quiz. She also passed the second. She can be confident she is going to pass the third.
 - d. People from that party are not in favor of the school administration's policies. She is from that party. So, she is not in favor of the policies, too.
2. Make a conjecture for each of the following items. Show your examples on which you base your conjecture.
 - a. The sign of the cube of a negative number
 - b. The sum of an even and an odd number (whether it is odd or even)
 - c. The product of an even and odd number (whether it is odd or even)
3. Answer the following questions.
 - a. Emily writes a few numbers on her notepad. These were 10, 2, 9, 3, 8, 4, 7, 5, 6, 6. What do you think will be the next number that Emily writes?
 - b. Mang Victor was able to sell 29 cups of *taho* on Monday, 35 cups on Tuesday, 41 cups on Wednesday, and 47 cups on Thursday. How many cups of *taho* did Mang Victor sell on Friday?



Example 2: Draw a conclusion based on the following statements:

*If a three-dimensional figure is a cube, then it has six square faces.
If a three-dimensional figure has six square faces, then it is a prism.*

Solution: Since the conclusion of the first conditional statement is the hypothesis of the second, the conclusion can be drawn by applying the Law of Syllogism. The Law of Syllogism states that if this is so, the hypothesis of the first statement implies that the conclusion of the second statement is true. Thus, the conclusion will be as follows:

Therefore, if a three-dimensional figure is a cube, then it is a prism.

Try It Yourself!



Draw a conclusion based on the following statements:

*If I pass the entrance exam, I will enroll in that school.
If I enroll in that school, I will have to stay in the dormitory.*

Example 3: Make a conclusion for the following:

*If the sum of the squares of the two legs of a right triangle is equal to the square of the longest side, then the triangle is a right triangle.
The sides of $\triangle ABC$ measure 3 cm, 4 cm, and 5 cm.*

Solution:

Step 1: Determine first if the second statement matches the hypothesis of the first statement.



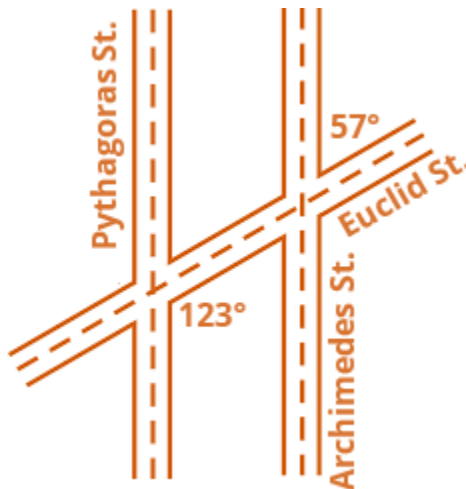
Check Your Understanding!

Construct a two-column direct proof for each of the following.

1. Given that $3x - 4 = -2x + 11$, prove that $x = 3$.
2. Prove that the product of an even number and any other number is even.
3. Given: $MT = AH$
Prove: $MA = TH$



4. Euclid Street cuts across Pythagoras Street and Archimedes Street, forming the angles shown in the picture. Prove that Pythagoras Street is parallel to Archimedes Street.



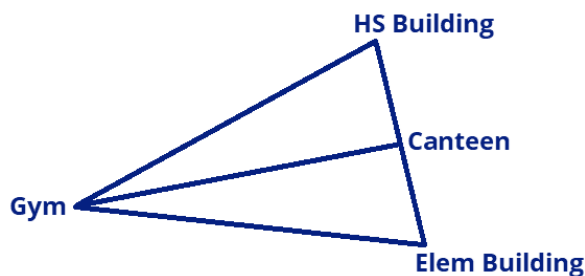
5. Ben sold half of his comic books and then bought 7 more. Now he has 11. Prove that Ben began with 8 comic books.



Check Your Understanding!

Construct a two-column indirect proof for each of the following.

1. Given that $3x > 12$, prove that $x > 4$.
2. Prove that the sum of two even numbers is an even number.
3. Prove that if n^2 divisible by 4, then n is even.
4. In a restaurant offering an eat-all-you-can promo, Jeff was able to eat 5 cups of rice. His wife was able to eat five less than twice the number of cups of rice Jeff ate. If his wife ate 3 cups of rice, prove that Jeff ate 4 cups of rice.
5. In a school campus, the elementary building and the high school building are of the same distance from the gym. The school canteen is situated exactly at the middle of the high school building and the elementary building. Prove using an indirect proof that the triangle formed by gym, canteen, and elementary building is congruent to the triangle formed by the gym, canteen, and high school building.





Key to *Let's Practice!*

Lesson 1

1. No
2. 1875
3. 2 097 152
4. 128 members

Lesson 2

1. Therefore, the sum of the measures of the angles of ABC is 180° .
2. Therefore, if I pass the entrance exam, I will have to stay in the dormitory.
3. Therefore, the hypotenuse of triangle XYZ is twice as long as its shortest side.
4. Therefore, if you can find a good graphic artist, more people will join the contest.

Lesson 3

1.

	Statement	Reason
1	$9x - 5 = 6x + 4$	Given
2	$9x - 6x - 5 + 5 = 6x - 6x + 4 + 5$	Addition Property of Equality (Add $-6x$ and 5 to both sides of the equation)
3	$3x = 9$	From statement 2
4	$(3x = 9) \left(\frac{1}{3}\right)$	Multiplication Property of Equality
5	$x = 3$	From statement 4

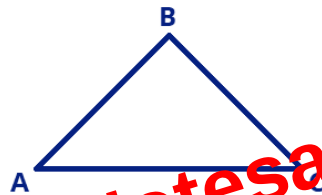
2.

	Statement	Reason
1	a and b are even numbers.	Given
2	$a = 2n$ and $b = 2m$; where m and n are any integers	Definition of even numbers



6	$4j^2 = 2k$	From statement 5
7	$n^2 = 2(2j^2) = 2k$ where j^2 is any integer	Factoring
8	Therefore, if n is even, then n^2 is an even number.	Due to contradiction made by statement 7, we conclude that the original statement is true.

4. Let the roof in Charles' drawing be represented by isosceles triangle ABC with $\angle A$ and $\angle C$ as its base angles.



Statement		Reason
1	$\triangle ABC$ is an isosceles triangle.	Given
2	$\angle B$ is the vertex angle.	Given (from the diagram).
3	$\angle A$ is not congruent to $\angle C$	Assumption
4	$AB = BC$	The legs of an isosceles triangle are congruent.
5	$\angle A \cong \angle C$	Angles opposite congruent sides of a triangle are congruent. (Contradiction to Statement 3.)

Since the last statement is a contradiction to Statement 3, we can say that the conclusion in the original statement is true, i.e. $\angle A \cong \angle C$.