Princeton University

Department of Mathematics

1 The Fast Track Introduction to Calculus

Chapter Preview. *Calculus is a new way of thinking about mathematics. This chapter provides you with a working understanding of the calculus mindset, core concepts of calculus, and the sorts of problems they help solve. The focus throughout is on the* ideas *behind calculus (the big picture of calculus); the subsequent chapters discuss the* math *of calculus. After reading this chapter, you will have an intuitive understanding of calculus that will ground your subsequent studies of the subject. Ready? Let's start the adventure!*

1.1 What Is Calculus?

Here's my two-part answer to that question:

Calculus is a mindset—a dynamics mindset. Content-wise, calculus is the mathematics of infinitesimal change.

Calculus as a Way of Thinking

The mathematics that precedes calculus—often called "pre-calculus," which includes algebra and geometry—largely focuses on *static* problems: problems devoid of change. By contrast, change is central to calculus—calculus is about *dynamics*. Example:

- What's the perimeter of a square of side length 2 feet? ⟵ *Pre-calculus problem.*
- How fast is the square's perimeter *changing* if its side length is *increasing* at the constant rate of 2 feet per second? ⟵ *Calculus problem.*

This statics versus dynamics distinction between pre-calculus and calculus runs even deeper—change is the *mindset* of calculus. The subject trains you to think of a problem in terms of dynamics (versus statics). Example:

- Find the volume of a sphere of radius r. *Pre-calculus mindset:* Use $\frac{4}{3}\pi r^3$ (Figure 1.1(a)).
- Find the volume of a sphere of radius r. *Calculus mindset:* Slice the sphere into a gazillion disks of tiny thickness and then add up their volumes (Figure 1.1(b)). When the disks' thickness is made "infinitesimally small" this approach reproduces the $\frac{4}{3}\pi r^3$ formula. (We will discuss why in Chapter 5.)