Example 5.2.3

Find approximately the volume of wood required to make a cubical box of edge length $6ft.$, using boards $\frac{1}{2}in.$ thick.

SOLUTION

The volume of cube of a length x is,

$V = x^3$, from which

$dV = 3x^3dx = 3x^3\Delta x$

Assume that $dx = \Delta x = \frac{0.5}{12}ft.$

$dV = 3(6ft.)\left(\frac{0.5}{12}ft.\right) = \mathbf{4.5ft.^3}$

Example 5.2.4

The diameter of sphere is measured and found to be $3ft.$ with a maximum error of $0.1in.$. Find the approximate maximum error in the computed volume.

SOLUTION

The volume of sphere of a radius r is,

$V = \frac{4}{3}\pi r^3$, from which

$dV = 4\pi r^2dr = 4\pi r^2\Delta r$

Assume that $maximun\ error = dr = \Delta r = 0.1\ in.$.

$dV = 4\pi(\frac{3}{2}(12))^2in.^2\ (0.1in.) = \mathbf{407.15\ in.^3}$

Propagated and Relative Error

If a measured value x is used to compute another value $f(x)$, the difference of $f(x+\Delta x)$ and $f(x)$ or Δy is the propagated error.

$$f(x+\Delta x) - f(x) = \Delta y \approx dy$$

Therefore we can use dy to estimate the propagated error.

If the propagated error is given in relative terms if we compare dV with V. The ratio $\frac{dV}{V}$ is called the relative error.

Example 5.2.5

The radius of the ball bearing is measured to be 0.8 inches. If the measurement is correct within 0.02 inches, estimate the a) propagated error and b) relative error.

SOLUTION

a) $V = \frac{4}{3}\pi r^3 \rightarrow dV = 4\pi r^2 dr$ where $r = 0.8in$

$dr = \pm 0.02$

$dV = 4\pi(0.8)^2(0.02)$

$= \pm\mathbf{0.1608\ in^3}$

b) $\frac{dV}{V} = \frac{4\pi r^2dr}{\frac{4}{3}\pi r^3} = \frac{3dr}{r} = \frac{3(\pm 0.02)}{0.8} = \pm\mathbf{0.075\ 0r\ 7.5}\%$

Example 5.2.6

A right circular cylinder has a height of 24 ft. and radius of 8 ft. If the error measuring is 3 inches, a) what is the possible error in volume b) what is the relative error?

SOLUTION

a). $V = \pi r^2 h$

$dV = \frac{\partial V}{\partial r}(dr) + \frac{\partial V}{\partial h}(dh)$

$dV = 2\pi rh(\pm 0.25ft) + \pi r^2(\pm 0.25ft)$

$dV = 2\pi(8ft)(3ft)(\pm 0.25ft) + \pi(8^2ft^2)(\pm 0.25ft)$

$\boldsymbol{dV = \pm 351.8\ ft^3}$

b).

For V,

$V = \pi r^2 h$

$V = \pi(8ft)^2(24ft) = 4825.5ft^3$

$Relative\ error = \frac{dV}{V}$

$Relative\ error = \frac{\pm 351.8\ ft^3}{4825.5\ ft^3} = \pm \mathbf{0.073\ or\ 7.3}\%$

4.2 Practice Problem

Answer the following problems:

1. Find approximately the volume of a thin spherical shell.
2. Find an approximate formula for the volume of a thin cylindrical shell of given height.
3. Use differentials to find the approximate value of $\sqrt{627}$.
4. Find the approximate error in computing the surface area of sphere having a diameter of $3ft$ with a maximum error of $0.01\ in.$.

Check your answer in the answer key. Please answer it all first before checking, this is important for self-assessment.

If you got 4 items correct, very well, you already understand the concept.

If you got 2 – 3 items correct, good, you understand most of the concepts, you only need to practice for a few times.

If you got 0 – 1, it is okay, do not worry. Compare your answer to the answer given, then try to answer the questions again before you proceed to the next lesson.

6.2 Exponential Growth and Decay

Exponential growth equation: $P(t) = P_o e^{kt}$

Exponential decay equation: $P(t) = P_o e^{-kt}$

for radioactive decay with half-life $t = 1/2$: $P(t) = P_o e^{-kt}$ with $k = \frac{ln2}{t_{\frac{1}{2}}}$

Example 5.6.5

The population of rabbits started $3\ years$ ago at 1000, but now has grown to $64{,}000$, then what will the population be $one\ year$ from now? Also, what is the total time it will take for the population to grow from 1000 to $400{,}000$?

SOLUTION:

Well, we have $P_o = 1000$, since that's the initial population. So the equation in the box above becomes $P(t) = 1000e^{kt}$. The problem is, we don't know what k is. We do know that $P = 64000$ when $t = 3$, so let's plug this in:

$$64000 = 1000e^{3k}$$
$$k = 2\ln 2$$

This means

$$P(4) = 1000e^{2ln2(4)} = \mathbf{256{,}000\ rabbits}$$

$$400{,}000 = 1000e^{2ln2(t)}$$
$$t = \frac{ln400}{2ln2}$$
$$\boldsymbol{t = 4.32\ years}$$

So although it takes $4\ years$ to get up to a population of $256{,}000$, it only takes approximately two-sevenths of a year more - about $3\frac{1}{2}$ - to get up to $400{,}000$. That's the power of exponential growth.

Answer Key

Answers for 5.1 Practice Problem:

1. $dy = 12x^3dx - 15x^2dx + dx$

2. $dz = 3(1-2v^3)(1-2v+v^4)^{-\frac{3}{2}}dv$

3. $du = (1-y)(2y-y^2)^{-\frac{1}{2}}dy$

4. $dx = \left(t^2 - \frac{1}{2\sqrt{t}} - \frac{2}{t^2}\right)dt$

Answer for 5.2 Practice Problem:

1. $surface\ area\ x\ thickness$

2. $circuference\ x\ height\ x\ thickness$

3. 25.04

4. $4.52in^2$

Answer for 4.3a Practice Problem:

1. $\frac{dy}{dx} = -\frac{2}{t}, \frac{d^2y}{dx^2} = -\frac{1}{t^3}$

2. $\frac{dy}{dx} = 6, \frac{d^2y}{dx^2} = 0$

3. $\frac{dy}{dx} = 1, \frac{d^2y}{dx^2} = 0$

4. $\frac{dy}{dx} = -\frac{2t^4-t^3}{2}, \frac{d^2y}{dx^2} = -(\frac{8t^3-3t^2}{2})(-\frac{t^3}{2})$

Answer for 4.3b Practice Problem:

1. $v = 60\ ft/sec, a =$ [illegible] $\frac{ft}{sec^2}, x = 28ft$

2. $1.5\ sec$

3. $v_x = 4; a_x = 2$

Answer for 5.4 Practice Problem:

1. $1/2$

2. 1

3. 0

4. $1/2$

Answer for 5.5 Practice Problem:

1. $1/2$

2. 1

3. $\frac{1}{4}k^3$

4. $54in.^3$