

Multivariable Calculus

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Real Numbers

- Real numbers can be positive, zero, or negative, and they can be rational or irrational.
- Denote the set of all real numbers as R .

Representation in 1 Dimension (R)

- Real numbers can be thought of as points on a line when an origin and unit length are chosen.
- Real numbers are also referred to as scalars.

Representation in 2 Dimensions (R^2)

- Pairs of real numbers $((x_1, x_2))$ form elements of R^2 .
- These elements label points in a plane when an origin and orthogonal axes are chosen.
- These elements are also called 2-dimensional vectors and can be represented as arrows.

Representation in 3 Dimensions (R^3)

- Triples of real numbers $((x_1, x_2, x_3))$ form elements of R^3 .
- These elements label points in space when an origin and three orthogonal axes are chosen.
- These elements are 3-dimensional vectors.

Generalizing to R^n

- R^n represents all n -tuples of real numbers $((x_1, x_2, \dots, x_n))$.
- These elements are points in n -dimensional space and are called n -dimensional vectors.

Vector Operations

- A scalar (α) can multiply a vector (x) to produce αx .
- Vectors (x and y) can be added ($x + y$) or subtracted ($x - y$).
- These operations define a vector space.

Geometric Interpretation

- Scalar multiplication (αx) changes the length and direction of a vector.
- Vector addition ($x + y$) can be visualized as completing a parallelogram or connecting the heads of vectors.
- Vector subtraction ($x - y$) can be visualized as completing a parallelogram or connecting the heads of vectors.