For some positive rational numbers $t_1 < 1$ and $t_2 < 1$ if $t_1 + t_2 = 1$ then

$$A^x + B^y = C^z$$

The A,B,C satisfying the above equation will have common prime factors.

3.Proof

From equation (6)

$$t_2 = 1 - t_1 \tag{8}$$

Let $t_1 = \frac{p_1^z}{q_1^x}$,where $p_1^z < q_1^x$ while p_1 and q_1 are natural numbers having no common factors. Similarly let $t_2 = \frac{p_2^z}{q_2^x}$,where $p_2^z < q_2^x$ while p_2 and q_2 are natural numbers having no common factors. Putting values of t_1 and t_2 in equation (3) and equation (4).

$$A^x = \frac{p_1^z}{q_1^x} C^z \tag{9}$$

$$B^y = \frac{p_2^z}{q_2^x} C^z \tag{10}$$

Putting value of t_1 in equation (8)

$$t_2 = 1 - \frac{p_1^z}{q_1^x} = \frac{q_1^x - p_1^z}{q_1^x} = \frac{h}{q_1^x} \tag{11}$$

Where $h = q_1^x - p_1^z$ is a natural number. From equation (4) and equation (11)

$$B^y = \frac{h}{q_1^x} C^z \tag{12}$$

Let $C = h^\alpha q_1^\beta$ for some natural numbers α and β

$$B^y = \frac{h}{q_1^x} (h^\alpha q_1^\beta)^z = h^{\alpha z + 1} q_1^{\beta z - x} \tag{13}$$

Le

$$\alpha z + 1 = my \tag{14}$$

For some natural number m. Similarly

$$\beta z - x = ny \tag{15}$$

For some natural number n. Putting equation (14) and equation (15) in equation (13)

$$B^y = h^{my} q_1^{ny} \tag{16}$$

Putting $C = h^\alpha q_1^\beta$ in equation (9)

$$A^x = \frac{p_1^z}{q_1^x} C^z = \frac{p_1^z}{q_1^x} (h^\alpha q_1^\beta)^z = p_1^z h^{\alpha z} q_1^{\beta z - x} \tag{17}$$

Let

$$z = k_1 x \tag{18}$$