E.g.: i. $\lim_{x\to 1}\left(\frac{x+2}{x+5}\right)=\frac{1+2}{1+5}=\frac{3}{6}=\frac{1}{2}$

ii. $\lim_{x\to 2}\left(\frac{x^2+2x-5}{3x+5}\right)=\frac{2^2+2\times 2-5}{3\times 2+5}=\frac{4+4-5}{6+5}=\frac{3}{11}$

iii. $\lim_{x\to -2}\left(\frac{x^2+3x+2}{x+3}\right)=\frac{(-2)^2+3\times(-2)+2}{-2+3}=\frac{4-6+2}{1}=\frac{0}{1}=0$

iv. $\lim_{x\to 3}(3x+2)(x^2+2x)=(3\times 3+2)(3^2+2\times 3)=(9+2)(9+6)=11\times 15=165$

Evaluation of $\lim_{x\to a}\frac{f(x)}{g(x)}$**, where** $\lim_{x\to a}f(x)=0$ and $\lim_{x\to a}g(x)=0$

$\lim_{x\to a}\frac{f(x)}{g(x)}=\frac{0}{0}$, when $\lim_{x\to a}f(x)=0$ and $\lim_{x\to a}g(x)=0$, is known as indeterminate form.

The form $\frac{0}{0}$ is called indeterminate form.

Note: The other indeterminate forms are $\frac{\infty}{\infty}$**,** $0\times\infty$**,** $\infty-\infty$**,** 0^0**,** 1^∞ **and** ∞^0**, etc**

We cannot find the limits such functions directly. The following methods are used to find the limits:

1. Factorization Method:

a) Factorize the numerator and denominator and cancel the common factors from the numerator and the denominator.

b) Be sure that the limit of the resulting denominator is non-zero.

c) Apply quotient rule of limit.

E.g.: Evaluate $\lim_{x\to 3}\frac{x^2-5x+6}{x-3}=\frac{0}{0}$

$$=\lim_{x\to 3}\frac{(x-2)(x-3)}{x-3}=\lim_{x\to 3}(x-2)=3-2=1$$

2. Substitution Method:

In this method, put $x=a+h$. As $x\to a$, $h\to 0$. Then $\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{h\to 0}\frac{f(a+h)}{g(a+h)}$. It can be simplified by cancelling the powers of h and can be simplified.

E.g.: Evaluate $\lim_{x\to\pi}\frac{\sin x}{\pi-x}=\frac{\sin\pi}{\pi-\pi}=\frac{0}{0}$

3. **Product rule**: If u and v are functions of x, then derivative of the product of two functions is equal to *first function x derivative of the second function + (plus) second function x derivative of the first function*.

i.e., $\mathbf{\frac{d}{dx}(uv) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)}$

E.g.: i. $y = e^{3x}\sin 4x$

$$\frac{dy}{dx} = e^{3x}\frac{d}{dx}(\sin 4x) + \sin 4x.\frac{d}{dx}\left(e^{3x}\right)$$

$$= e^{3x}.\cos 4x.\ 4 + \sin 4x.e^{3x}.3 = e^{3x}(4\cos 4x + 3\sin 4x)$$

ii. $y = x^2\tan x$

$$\frac{dy}{dx} = x^2\frac{d}{dx}(\tan x) + \tan x\frac{d}{dx}\left(x^2\right)$$

$$= x^2\sec^2 x + \tan x.2x = x^2\sec^2 x + 2x\tan x$$

Corollary of product rule:

If u, v and w are functions of x, then $\mathbf{\frac{d}{dx}(uvw) = uv.\frac{d}{dx}(w) + vw.\frac{d}{dx}(u) + uw.\frac{d}{dx}(v)}$

E.g.: $y = x^2e^x\tan x$

$$\frac{dy}{dx} = x^2e^x\frac{d}{dx}(\tan x) + e^x\tan x\frac{d}{dx}\left(x^2\right) + x^2\tan x\frac{d}{dx}\left(e^x\right)$$

$$= x^2e^x\sec^2 x + e^x\tan x.2x + x^2\tan x.e^x$$

$$= xe^x\left(x\sec^2 x + 2\tan x. + x\tan x.\right) = xe^x\left(x\sec^2 x + (2+x)\tan x.\right)$$

4. **QUOTIENT FORMULA**: If u and v are any two functions of x, then quotient of two functions is equal to (2^{nd} function x derivative of the 1^{st} function minus 1^{st} function x derivative of the 2^{nd} function) divided by square of the 2^{nd} function.

i.e., $\mathbf{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{v^2}}$

E.g.: $y = \dfrac{Sin\,x + Cos\,x}{Sin\,x - Cos\,x}$.

$$\frac{dy}{dx} = \frac{(\sin x - \cos x).\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x)\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$