Proof of Theorem ⊛ using CMVT

If $\lim\limits_{x \to x_0^+} \dfrac{f'(x)}{g'(x)} = \ell$ then $\lim\limits_{x \to x_0^+} \dfrac{f(x)}{g(x)} = \ell$.

<u>Proof</u> Choose $(x_n)_{n \geq 1}$ such that $x_n \to x_0$ with $x_n > x_0$

Apply CMVT on interval $[x_0, x_n]$

$\exists\, x_0 < c_n < x_n$ $\quad \dfrac{f'(c_n)}{g'(c_n)} = \dfrac{f(x_n) - \cancel{f(x_0)}}{g(x_n) - \cancel{g(x_0)}}$

So $\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)} = \lim\limits_{n \to \infty} \dfrac{f(x_n)}{g(x_n)} = \lim\limits_{n \to \infty} \dfrac{f'(c_n)}{g'(c_n)} = \dfrac{f'(x_0)}{g'(x_0)}$

$c_n \to x_0$ by Pinching Thm.