$$\vec{b}\times 2\vec{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 2 \\ 0 & 2 & 0 \end{vmatrix} = -4\hat{i} + 2\alpha\hat{k}$$

$$\therefore \left|\vec{b}\times 2\hat{j}\right| = \sqrt{16+4\alpha^2} = \sqrt{16+4\times\frac{9}{4}} = 5$$

18. $|\vec{a}| = 9$ and $\left(x\vec{a}+y\vec{b}\right)\cdot\left(6y\vec{a}-18x\vec{b}\right) = 0$

$$\Rightarrow 6xy|\vec{a}|^2 - 18x^2\left(\vec{a}\cdot\vec{b}\right) + 6y^2\left(\vec{a}\cdot\vec{b}\right) - 18xy\left|\vec{b}\right|^2 = 0$$

$$\Rightarrow 6xy\left(|\vec{a}|^2 - 3\left|\vec{b}\right|^2\right) + \left(\vec{a}\cdot\vec{b}\right)\left(y^2-3x^2\right) = 0$$

This should hold $\forall x, y \in R\times R$

$$\therefore |\vec{a}|^2 = 3|b|^2 \text{ and } \left(\vec{a}\cdot\vec{b}\right) = 0$$

Now $\left|\vec{a}\times\vec{b}\right|^2 = |\vec{a}|^2\left|\vec{b}\right|^2 - \left(\vec{a}\cdot\vec{b}\right)^2$

$$= |\vec{a}|^2\cdot\frac{|\vec{a}|^2}{3}$$

$$\therefore \left|\vec{a}\times\vec{b}\right| = \frac{|\vec{a}|^2}{\sqrt{3}} = \frac{81}{\sqrt{3}} = 27\sqrt{3}$$

19. For angle to be acute

$\vec{u}\cdot\vec{v} > 0$

$$\Rightarrow a\left(\log_e b\right)^2 - 12 + 6a\left(\log_e b\right) > 0$$

$\forall b > 1$

Let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$y = at^2 + 6at - 12$ and $y > 0\ \forall t > 0$

$\Rightarrow a \in \phi$

20. $\vec{a} = 3\hat{i}+\hat{j}, \vec{b} = \hat{i}+2\hat{j}+\hat{k}$

As $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}+\lambda\vec{c}$

$$\Rightarrow \vec{a}\cdot\vec{c}\left(\vec{b}\right) - \left(\vec{a}\cdot\vec{b}\right)\vec{c} = \vec{b}+\lambda\vec{c}$$

$$\Rightarrow \vec{a}\cdot\vec{c} = 1, \vec{a}\cdot\vec{b} = -\lambda$$

$$\Rightarrow \left(3\hat{i}+\hat{j}\right)\cdot\left(\hat{i}+2\hat{j}+\hat{k}\right) = -\lambda$$

$$\Rightarrow \lambda = -5$$

21. $\hat{a}\wedge\hat{b} = \frac{\pi}{4} = \phi$

$$\hat{a}\cdot\hat{b} = |\hat{a}|\left|\hat{b}\right|\cos\phi$$

$$\hat{a}\cdot\hat{b} = \cos\phi = \frac{1}{\sqrt{2}}$$

$= 90 + 27\sqrt{2}$

22. $|\vec{a}||\vec{b}||\vec{c}| = 14$

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \dfrac{2\pi}{3}$

So, $\vec{a} \cdot \vec{b} = -\dfrac{1}{2}ab, \vec{b} \cdot \vec{c} = -\dfrac{1}{2}bc, \vec{a} \cdot \vec{c} = -\dfrac{1}{2}ac$

(let)

$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{b})$

$= \dfrac{1}{4}ab^2c + \dfrac{1}{2}ab^2c = \dfrac{3}{4}ab^2c$

Similarly

$(\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \dfrac{3}{4}abc^2$

$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \dfrac{3}{4}a^2bc$

$168 = \dfrac{3}{4}abc(a + b + c)$

So, $(a + b + c) = 16$

23. $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$

$\Rightarrow 13 - 1 - 4\lambda = 0 \Rightarrow \lambda = 3$

$\Rightarrow \vec{b} = \hat{i} + \hat{j} + 3\hat{k} \Rightarrow \vec{a} \times \vec{b} = 13\hat{i} - \hat{j} - 4\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) \times \vec{b} = (13\hat{i} - \hat{j} - 4\hat{k}) \times (\hat{i} + \hat{j} + 3\hat{k})$

$\Rightarrow -21\vec{b} - 11\vec{a} = \vec{i} - 43\hat{j} + 14\hat{k}$

$\Rightarrow \vec{a} = -2\hat{i} - 2\hat{j} - 7\hat{k}$

Now $(\vec{b} - \vec{a}) \cdot (\hat{k} - \hat{j}) + (\vec{b} + \vec{a}) \cdot (\hat{i} - \hat{k}) = 14$

24. $\bar{a} \cdot \bar{c} = 5 \Rightarrow 2c_1 + c_2 + 3c_3 = 5 \quad ...(1)$

$\bar{b} \cdot \bar{c} = 0 \Rightarrow 3c_1 + 3c_2 + c_3 = 0 \quad ...(2)$

And $[\bar{a}\ \bar{b}\ \bar{c}] = 0 \Rightarrow \begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 8c_1 - 7c_2 - 3c_3 = 0 \quad ...(3)$

By solving $(1),(2),(3)$ we get

$c_1 = \dfrac{10}{122}, c_2 = \dfrac{-85}{122}, c_3 = \dfrac{225}{122}$

$\therefore 122(c_1 + c_2 + c_3) = 150$

25. $|\bar{a} + \bar{b}| = \sqrt{3}, |\bar{a} + \bar{b}|^2 = 3 \Rightarrow \vec{a} \cdot \vec{b} = \dfrac{1}{2}$