Determining the antiderivative of a function is a matter of evaluating the infinite sum of infinitesimal terms, the infinite series, that an integral represents. Fortunately, all of the hard work has been done for us my the mathematicians, so here, in your physics course, we can provide you with a few simple rules for arriving at the results. If you haven't already done so, you will learn where these simple rules come from in your calculus class. Here, we simply present them to you, along with some information on notation and usage, without proof.

First a few comments on the relation between the derivative operator and the integral operator. We again rely on the example of the object whose velocity is given by $v(t) = 1.5\frac{\text{m}}{\text{s}^3}t^2$ to make our points. First off, as you know, the velocity $v(t)$ is just the time derivative of the position variable x: $v(t) = \frac{dx}{dt}$. This means that our integral $\int_0^{4.0\text{s}} v(t)\,dt$ is the same thing as $\int_0^{4.0\text{s}} \frac{dx}{dt}\,dt$. We can treat this expression as if the dt's cancel and write it as $\int_0^{4.0\text{s}} dx$. We don't need any fancy rules to interpret this. It represents the sum of all the infinitesimal changes in position dx that the object experiences from time $t = 0$ to time $t = 4.0\text{s}$. This is nothing but the total change in position of the object, which we can express as $x(4.0\text{s}) - x(0)$. So the antiderivative function is just our position x. (But see footnote 3.) Check it out. Start with x. Take the derivative of x with respect to t, $\frac{dx}{dt}$. Now integrate that. You get x back. Integration is the inverse operation to taking the derivative. It works the other way too.

If you integrate a function, and then take the derivative of the result, you get the original function back. So when you integrate a function, what you are doing is finding a function whose derivative is equal to the original function. Here, when we say "integrate a function" we really mean "find the antiderivative of a function." Such an integral is called an indefinite integral and is written without limits of integration. (As you might guess, an integral that includes the limits of integration is called a definite integral.) Let's use this information to arrive at an answer for the example we have been talking about.

We need to calculate $\int_0^{4.0\text{s}} v(t)\,dt$ for the case in which $v(t) = 1.5\frac{\text{m}}{\text{s}^3}t^2$. That is, we need to calculate $\int_0^{4.0\text{s}} 1.5\frac{\text{m}}{\text{s}^3}t^2\,dt$. Now this represents an infinite sum in which every term is being multiplied by the constant $1.5\frac{\text{m}}{\text{s}^3}$. We can factor that constant out of the sum and write the

[3] The position function is actually the initial position plus the antiderivative. Here we specialize to the case of an initial position of zero. In general, when you find an antiderivative of $f(x)$ you are finding a function $g(x)$ whose derivative is $f(x)$. Add any constant to $g(x)$ that you want. Call the result $h(x) = g(x) + \text{constant}$. $h(x)$ must also be an antiderivative of $f(x)$ because the derivative of $h(x)$ is the derivative of g(x) plus the derivative of the constant (which is of course 0). So if the derivative of $g(x)$ is $f(x)$ then the derivative of $h(x) = g(x) + \text{constant}$ is also $f(x)$. That means that $f(x)$ has an infinite set of antiderivatives, one for each of the infinite number of possible values of the constant.