VI. Separation Technique (1st order)

a. All 1st order separable D.E. can be solved

i. Get all x variables on one side, and all y on other

ii. dx with x and dy with y

iii. integrate and simplify for solution

- Solutions may be explicit or implicit

* Review: Synthetic Division

$$\frac{x+3}{x+7} \quad \xrightarrow{\text{flip}} \quad -7 \;\Big|\; \begin{array}{rr} 1 & 3 \\ \downarrow & -7 \\ \hline 1 & -4 \end{array}$$

$$\boxed{1 - \frac{4}{x+7}}$$

* Partial Fractions sometimes may be required

$$y(y-1)\cdot\frac{1}{\cancel{y(y-1)}} = \left[\frac{A}{y} + \frac{B}{y-1}\right]\cdot y(y-1)$$

$$1 = \frac{A\cdot \cancel{y}(y-1)}{\cancel{y}} + \frac{B\cdot y\cancel{(y-1)}}{\cancel{(y-1)}}$$

$$1 = A(y-1) + By$$

LINEAR EQS.

VII. Method for non-separable 1st order Linear D.E.

a. 4 Step Process

i. Write equation in standard form

$$\rightarrow \frac{dy}{dx} + P(x)y = Q(x)$$

ii. Find the integrated factor, $\mu(x)$

$$\rightarrow \mu(x) = e^{\int P(x)dx}$$

iii. Multiply the standard form D.E. by the integrated factor

$$\rightarrow \mu(x)\frac{dy}{dx} + \mu(x)P(x)y = \mu(x)Q(x)$$

iiii. Rewrite in form $\frac{d}{dx}[\mu(x)y] = \mu(x)Q(x)$

Note: Check and be sure coefficient of y is the derivative of $\mu(x)$

IV. Method of Undetermined Coefficients – Superposition Approach

a. (Non-Homogeneous, Linear D.E.)

look at R.H.S. and assume y_p is most basic form

ex: R.H.S. $= 4t^2$ assume $y_p = At^2 + Bt + C$

then get derivatives and plug into D.E. (simplify)

Equate coefficients and constants

ex: if R.H.S. $= -8e^x\cos(2x)$ then $y_p = e^x\left[A\cos 2x + B\sin(2x)\right]$

whenever sin or cos you include both

b. Easier Way (According to Professor)

- Get homogeneous part first $y_h = \ldots$ get roots & get form for y_h

- if R.H.S. of D.E. is not a solution to ... then $s=0$
- if it is $s=1$, if appears twice $s=2$ ... for n^{th} order number of roots

$f(x) =$

$f(x)$	Form of y_p
$a_n x^n + a_1 n + a_0$	$y_p = x^s\left[A_n x^n + \ldots A_1 x + A_0\right]$ ← diff. constants
$ae^{\alpha x}$	$y_p = x^s\left[Ae^{\alpha x}\right]$
$\cos(Bx)$ or $\sin(Bx)$	$y_p = x^s\left[A\cos Bx + B\sin Bx\right]$
$e^{\alpha x}\cos(Bx)$ or $e^{\alpha x}\sin(Bx)$	$y_p = x^s\left[Ae^{\alpha x}\cos Bx + Be^{\alpha x}\sin Bx\right]$
$P_n(x)e^{\alpha x}\cos(Bx) + Q_n(x)e^{\alpha x}\sin(Bx)$	$y_p = x^s\left[P_N(x)e^{\alpha x}\cos Bx + Q_N(x)e^{\alpha x}\sin Bx\right]$

With trig. functions → will only work with sin or cos, (no ratios of them)

IVX. Eigenvalues & Eigenfunctions

CASE 1, $\lambda = 0$ (Trivial solution)

CASE 2, $\lambda < 0$ (Trivial solution)

CASE 3, $\lambda > 0$ (Gives solution)

Get to either $\cos(\text{—})$ or $\sin(\text{—})$

Eigenvalues is $\lambda_n = \cdots$ solve for λ

Eigenfunction is when you plug everything back into $y(x)$ or (y_n)