Tuesday, Feb 20, 2024

Permutations and Factorials

Simplify: $\frac{10!}{7!} = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{7\times6\times5\times4\times3\times2\times1}$

$= 10\times9\times8$

$= 720$

b) $\frac{12!}{10!} = \frac{12\times11\times10\times9\cdots\times1}{10\times9\cdots\times1}$

$= 12\times11$

$= 132$

c) $\frac{n!}{(n-1)!} = \frac{n(n-1)(n-2)\times\cdots\times3\times2\times1}{(n-1)(n-2)\times\cdots\times3\times2\times1}$

$= n$

d) $\frac{(n+3)!}{(n+1)!} = \frac{(n+3)(n+2)(n+1)\times\cdots\times3\times2\times1}{(n+1)(n)(n-1)\times\cdots\times3\times2\times1}$

$= (n+3)(n+2)$

$= n^2+2n+3n+6$

$= n^2+5n+6$

1. 6 letter words from TRAVEL (letters cannot be repeated)

a) with all 6 letters: $6! = 720$

b) first letter must be L: $\underline{1}\times\underline{5}\times\underline{4}\times\underline{3}\times\underline{2}\times\underline{1} = 1\times5! = 120$

c) second and fourth letters are vowels

$\underline{4}\ \underline{2}\ \underline{3}\ \underline{1}\ \underline{2}\ \underline{1}$ (V V) vowels = A, E $2\times1\times4! = 48$

d) A and V must be adjacent: treat "AV" as one letter: $\underline{5}\ \underline{4}\ \underline{2}\ \underline{3}\ \underline{2}\ \underline{1}$ (AV) $5!\times2! = 120\times2 = 240$

2. algebra books: 5 physics books: 7

five algebra books: 5!

since algebra books are kept together, they will be treated as one item

so, physics + algebra books: $(7+1)+8$

total number of arrangements $= 8!\times5!$

$= 40320\times120$

$= 4,838,400$

3. boys: 6! no two people can sit together

girls: 6! $6!\times6!\times2 = 1,036,800$

4. MATHPOWER

a) all letters, no restrictions

$9! = 362,880$

b) the first letter must be P and last letter an R

$\underline{P}\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \underline{R}$ $1\times7!\times1 = 5040$

c) the letters MATH must be together

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ (MATH) $= 4!\times5! = 2880$

d) the letters MATH must be together in that order

$\underline{MATH}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 1\times5! = 120$ treat "MATH" as one word

5. $4, 5$ no digit can repeat

$4! + 4P_3 + 4P_2 + 4P_1 = 24+24+12+4 = 64$

$4P_3 = \frac{4!}{(4-3)!} = \frac{24}{1} = 24$

$4P_2 = \frac{4!}{(4-2)!} = \frac{24}{2} = 12$

$4P_1 = \frac{4!}{(4-1)!} = \frac{4!}{3!} = \frac{24}{6} = 4$

6. _ _ _ _ _ _ _ _ _

$9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{362880}{120} = 3024$

7. MINIMUM

$\frac{n!}{a!b!c!} = \frac{7!}{3!2!} = \frac{5040}{6(2)} = \frac{5040}{12} = 420$

8. 6 different flags for signaling. 4 flags required

MINIMUM of 4 flags

4 flags: $6\times5\times4\times3 = 360$ Total: $360+720+720 = 1800$

5 flags: $6\times5\times4\times3\times2 = 720$ or do permutation:

6 flags: $6\times5\times4\times3\times2\times1 = 720$ $6P_4 + 6P_5 + 6P_6$

$6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = 360$

$6P_5 = \frac{6!}{(6-5)!} = \frac{6!}{1!} = 720$

$6P_6 = \frac{6!}{(6-6)!} = \frac{720}{1} = 720$

Permutations with Restrictions

ENGINEERING: $\frac{n!}{a!b!c!}$

$= \frac{11!}{3!3!2!2!}$

$= \frac{39916800}{144} = 277,200$