Calculate following limits:

37. $\lim\limits_{x\to\frac{1}{2}\pm 0}\dfrac{3x+1}{2x-1}$. **38.** $\lim\limits_{x\to-3\pm0}\dfrac{1}{x+3}$. **39.** $\lim\limits_{x\to1+0}\ln(x-1)$.

40. $\lim\limits_{x\to\pm\infty}\left(\dfrac{1}{2}\right)^x$. **41.** $\lim\limits_{x\to2}\dfrac{x^2-x-2}{x-2}$. **42.** $\lim\limits_{n\to\infty}\dfrac{5n+3}{3n-2}$.

43. $\lim\limits_{n\to\infty}\left(\dfrac{n+1}{2n}\right)^n$. **44.** $\lim\limits_{x\to-\infty}\dfrac{\sqrt{4x^2+1}}{x+1}$. **45.** $\lim\limits_{n\to\infty}\left(\dfrac{5n+4}{3n}\right)^{2n}$.

46. $\lim\limits_{x\to-1}\dfrac{x^2-x-2}{x^3+1}$. **47.** $\lim\limits_{x\to+\infty}\dfrac{1-5x}{\sqrt{4x^2+4x+1}}$. **48.** $\lim\limits_{x\to2}\dfrac{-4x^2+5x+6}{x^2-3x+2}$.

49. $\lim\limits_{x\to1}\dfrac{x-1}{\sqrt{x+3}-2}$. **50.** $\lim\limits_{n\to\infty}\dfrac{5^n+3^n}{5^{n-2}+3^{n-1}}$. **51.** $\lim\limits_{x\to2}\left(\dfrac{1}{x-2}-\dfrac{12}{x^3-8}\right)$.

52. Find points of discontinuity: a) $y=\begin{cases}\dfrac{1}{x-1}, & x\le 0;\\ \dfrac{1}{x}, & x>0;\end{cases}$ б) $y=\dfrac{x^2-4}{x+2}$.

The solution of typical problems

Problem 1. Calculate $\lim\limits_{x\to1}\dfrac{3x+5}{4x-2}$.

◁ Since $f(x)=\dfrac{3x+5}{4x-2}$ is elementary function, then it is continuous in its domain $D_f=\left(-\infty;\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2};+\infty\right)$. The point $x_0=1\in D_f$, then by definition of continuous function $\lim\limits_{x\to1}\dfrac{3x+5}{4x-2}=f(1)=\dfrac{3\cdot1+5}{4\cdot1-2}=4$.▷

In order to calculate an indeterminate form $\left(\dfrac{0}{0}\right)$ when $x\to x_0$ we need to cancel a fraction on $(x-x_0)^s$, where $s>0$ (often $s=1$). If irrationality interferes with a given reduction, it is first eliminated.

Problem 2. Calculate $\lim\limits_{x\to4}\dfrac{x^2-16}{x^2-7x+12}$.

◁ Convert the expression $n-\sqrt{n^2-n}=\dfrac{\left(n-\sqrt{n^2-n}\right)\left(n+\sqrt{n^2-n}\right)}{n+\sqrt{n^2-n}}=$

$\dfrac{n}{n+\sqrt{n^2-n}}$. Then $\lim\limits_{n\to\infty}\left(n-\sqrt{n^2-n}\right)(\infty-\infty)=\lim\limits_{n\to\infty}\dfrac{n}{n+\sqrt{n^2-n}}\left(\dfrac{\infty}{\infty}\right)=\lim\limits_{n\to\infty}$

$\dfrac{n}{n\left(1+\sqrt{1-\dfrac{1}{n}}\right)}==\lim\limits_{n\to\infty}\dfrac{1}{1+\sqrt{1-\dfrac{1}{n}}}=\dfrac{1}{2}$. ▷

Problem 8. Find points of discontinuity for the piecewise function $f(x)=\begin{cases}x^2, & x\le 1;\\ 2x, & x>1.\end{cases}$

◁ Find the one sided limits of the function at $x\to 1\pm 0$:

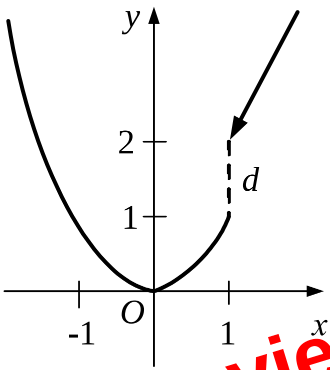

Рис.33

$$f(1-0)=\lim_{x\to 1-0}f(x)=\lim_{x\to 1-0}x^2=1,$$

$$f(1+0)=\lim_{x\to 1+0}f(x)=\lim_{x\to 1+0}2x=2,\quad f(1)=1.$$

Because one-sided limits are finite but not equal, then in the point $x_0=1$ the function has the jump

$d=f(1+0)-f(1-0)=1$. ▷

Answers

1. 1. **2.** 0. **3.** 1. **4.** ∞. **5.** 0. **6.** $\pm\infty$. **7.** 0. **8.** 4. **9.** 3,5. **10.** $7/6$. **11.** $2/3$.

12. $-1/56$. **13.** 0,4. **14.** –0,25. **15.** $+\infty$. **16.** 0. **17.** $+\infty$. **18.** 1. **19.** 0. **20.** $1/6$. **21.** 1) б; 2) а; 3) в. **22.** 1) в; 2) а; 3) б. **23.** No. **24.** Yes. **25.** a) $x=1$; b) $x=2$; c) $x=-2$; $x=2$. **26.** –0,5. **27.** 0. **28.** 0,5. **29.** 0. **30.** ∞. **31.** 0,2. **32.** ∞. **33.** 0. **34.** 0,5. **35.** 0,5. **36.** $x=-1$(jump). **37.** ∞. **38.** $\pm\infty$. **39.** $-\infty$. **40.** 0; $+\infty$. **41.** 3. **42.** $5/3$. **43.** 0. **44.** –2. **45.** $+\infty$. **46.** –1. **47.** –2,5. **48.** –11. **49.** 4. **50.** 25. **51.** 0,5. **52.** a) $x=0$ (asymptotic discontinuity); б) $x=-2$ (removable)

Practical class №15. Special limits

The first special limit: $\lim\limits_{x\to 0}\dfrac{\sin x}{x}\left(\dfrac{0}{0}\right)=1,$

its conclusions: