$\int \left(\frac{1}{\sqrt{25-u^2}}\right).\frac{1}{2}du$

Pulling out ½

$\frac{1}{2}\int \left(\frac{1}{\sqrt{25-u^2}}\right)du$

Integrating by trigonometric substitution

Recall, $\int \left(\frac{1}{\sqrt{a^2-x^2}}\right)dx = \ sin^{-1}\left(\frac{x}{a}\right)+c$

Re-writing the Integral in the right pattern for the trigonometric substitution

$\frac{1}{2}\int \left(\frac{1}{\sqrt{5^2-u^2}}\right)du$

x= u

a=5

$\frac{1}{2}sin^{-1}\left(\frac{u}{a}\right)+c$

But, u= 2x-4

$\int \left(\frac{1}{\sqrt{9+16x-4x^2}}\right)dx = \frac{1}{2}sin^{-1}\left(\frac{2x-4}{5}\right)+c$

<u>CASE 2</u>

Integrals in the form $\int \left(\frac{AX+B}{(ax^2+bx+c)^n}\right)dx, where\ \mathrm{n}\ \ is\ a\ positive\ integer\ and\ where$

$\frac{d}{dx}(ax^2+bx+c) \neq Ax+B, can\ be\ solved\ by\ splitting\ it\ into\ simpler\ \ integrals$

that are in standard form.

<u>WORKED EXAMPLE 1:</u>

Find $\int \left(\frac{2x+3}{9x^2+6x+5}\right)dx$

In this case, $\frac{d}{dx}(9x^2+6x+5) = 18x+6.$

x= u

a=2

$\frac{7}{9}\left[\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right]+c$

But, U=3x+1

Further simplifying

$\frac{7}{18}\tan^{-1}\left(\frac{3x+1}{2}\right)+c_2$

$\int\left(\frac{\frac{7}{3}}{9x^2+6x+5}\right)dx=\frac{7}{18}\tan^{-1}\left(\frac{3x+1}{2}\right)+c_2$

Therefore,

$\int\left(\frac{2x+1}{9x^2+6x+5}\right)dx=\frac{1}{9}\ln(9x^2+6x+5)+\frac{7}{18}\tan^{-1}\left(\frac{3x+1}{2}\right)+C$

NOTE: c_1 *and* c_2 *have been replaced with a single constant* C

<u>WORKED EXAMPLE 2</u>: Find $\int\left(\frac{(x+1)}{(x^2+4x+5)^2}\right)dx$

In this case, $\frac{d}{dx}(x^2+4x+5)=2x+4$

We would need to re-write x+1 as a constant multiple of 2x+4

x+1=P(2x+4)+Q

Further simplifying

x+1=2Px+4P+Q

Equating corresponding coefficients

2P=1…..(I)

4P+Q=1…..(II)

Solving simultaneously