Matrices and Determinants

1. i) $|A| \neq 0$ – Sol^n unique

ii) $|A| = 0$, $(\text{Adj } A)B = 0$, it has infinitely many sol^n
System = Consistant.

iii) $|A| = 0$, $(\text{Adj } A)B \neq 0$
System = inconsistant.

2. $|\text{Adj } A| = |A|^{n-1}$

3. $|\text{Adj}(\text{Adj } A)| = |A|^{(n-1)^2}$

4. $A^{-1} = \frac{\text{Adj } A}{|A|}$

5. $A(\text{Adj } A) = |A| I_n$

6. $|A^{-1}| = \frac{1}{|A|}$

7. $|AB| = |A||B|$

8. If A is Singular Matrix
$|A| = 0$

9. $\text{Adj}(\text{Adj } A) = |A|^{n-2} \cdot A$

10. $|kA| = k^n |A|$

11. $|A| = |A^T|$

12. $A + A^T \rightarrow$ Symmetric

13. $A - A^T \rightarrow$ Skew symmetric

14. $AA^T =$ Symmetric

15. $AA^{-1} = I$

16. $(AB)^{-1} = B^{-1}A^{-1}$

17. $(A^T)^{-1} = (A^{-1})^T$

18. If A is a Skew Symmetric Matrix then all the diagonals are zero.

Proof :- $A_{ij} = -A_{ji}$
$i = j$
$A_{ii} = -A_{ii}$
$A_{ii} + A_{ii} = 0$
$2A_{ii} = 0$
$A_{ii} = 0$