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corresponds to the equation

$$0x_1 + 0x_2 + 0x_3 = -1$$

Obviously, there are no numbers x_1, x_2, x_3 that satisfy this equation, and therefore, the linear system is inconsistent, i.e., it has no solution. In general, if we obtain a row in an **augmented matrix** of the form

$$[0 \ 0 \ 0 \ \cdots \ 0 \ c]$$

where c is a **nonzero** number, then the linear system is inconsistent. We will call this type of row an **inconsistent row**. However, a row of the form

$$[0 \ 1 \ 0 \ 0 \ 0]$$

corresponds to the equation $x_2 = 0$ which is perfectly valid. □

1.4 Geometric interpretation of the solution set

The set of points (x_1, x_2) that satisfy the linear system

$$\begin{aligned} x_1 - 2x_2 &= -1 \\ -x_1 + 3x_2 &= 3 \end{aligned} \tag{1.2}$$

is the intersection of the two lines determined by the equations of the system. The solution for this system is $(3, 2)$. The two lines intersect at the point $(x_1, x_2) = (3, 2)$, see Figure 1.1.

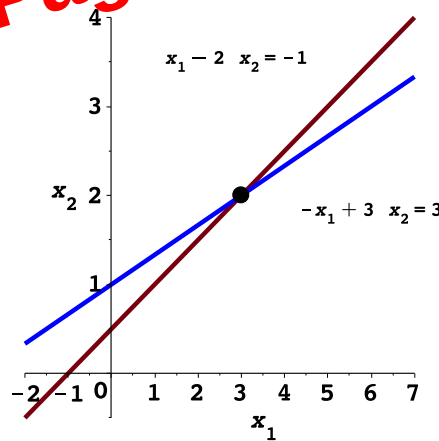


Figure 1.1: The intersection point of the two lines is the solution of the linear system (1.2)

Similarly, the solution of the linear system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 2x_2 - 8x_3 &= 8 \\ -4x_1 + 5x_2 + 9x_3 &= -9 \end{aligned} \tag{1.3}$$

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