2- Exact method:-

exact form:

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad F_x = M, \; F_y = N$$

→ If $M_y = N_x$, then it's exact.

Solving method:-

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$M_y = N_x$$

→ $\int M(x,y)\,dx + \int N(x,y)\,dy = C$

→ repeated terms are written once.

Ex:

* $\frac{dy}{dx} = \frac{4xy^3 + \cos x}{2y + 6x^2y^2}$

→ $(4xy^3 + \cos x)\,dx + (2y + 6x^2y^2)\,dy = 0$

$M_y = 12xy^2 = N_x = 12xy^2$

$\int 4xy^3 + \cos x \, dx + \int 2y + 6x^2y^2\,dy = 0$

$2x^2y^3 + \sin x + y^2 = C$

* $\int 2x + \sin y - ye^{-x}\,dx + \int x\cos y + \cos y + e^{-x}\,dy = 0$

$M_y = \cos y - e^{-x} \quad N_x = \cos y - e^{-x}$

~~$\int 2x + \sin y - ye^{-x}$~~

$x^2 + \cancel{x\sin y} + \cancel{ye^{-x}} + \cancel{x\sin y} + \sin y + ye^{-x} = 0$

→ $x^2 + x\sin y + \sin y + ye^{-x} = 0$

* $\int y x^{y-1}\,dx + \int x^y \ln x\,dy = 0$

$M_y = x^{y-1} + yx^{y-1}\ln x \qquad N_x = yx^{y-1}\ln x + x^{-1}x^y$

→ $\frac{yx^{y}}{y} + \frac{x^y}{\cancel{\ln x}}\cancel{\ln x} = C$

→ x^y

Special Cases:-

- Integrating factor:- if $M_y \neq N_x$, so the equation may be multiplied or divided by (x,y)