Here, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ are the three medians of ΔABC.

The medians of a triangle always lie inside the triangle.

From the figure, it can be observed that the medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect each other at a common point G.

"The point of intersection of the medians is called the **centroid** of the triangle."

Thus, medians of a triangle are concurrent.

The point where medians intersect each other is known as the point of concurrence.

In the above given figure, G is the point of concurrence.

Construction of Median of Triangle

1. Draw a △ABC.
2. With B and C as centres and radius more than half of BC, draw two arcs intersecting at points X and Y. Join XY thus meeting the line BC at point P.

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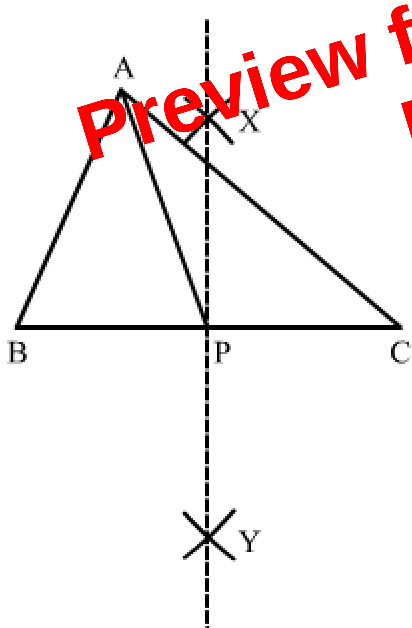
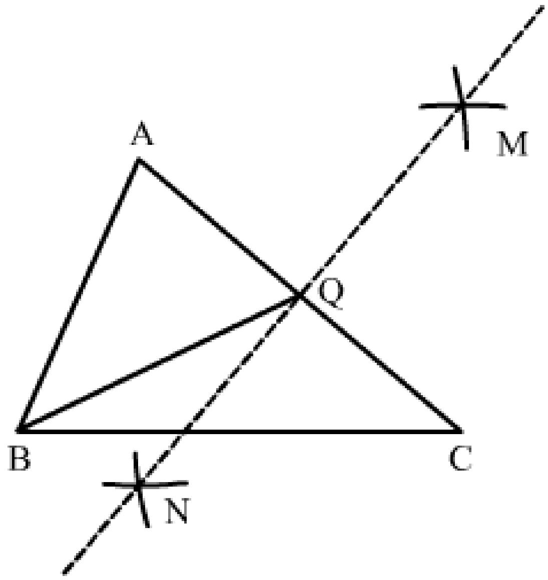
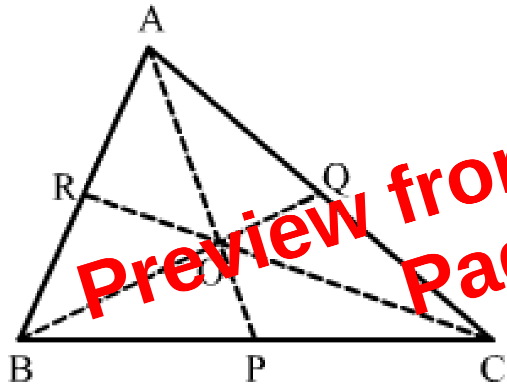


3. With A and C as centres and radius more than half of AC, draw two arcs intersecting at points M and N. Join MN thus meeting the line AC at point Q.



4. Similarly, draw the perpendicular bisector of line AB meeting AB at point R.
5. Join AP, BQ and CR. Let the meeting point be O.



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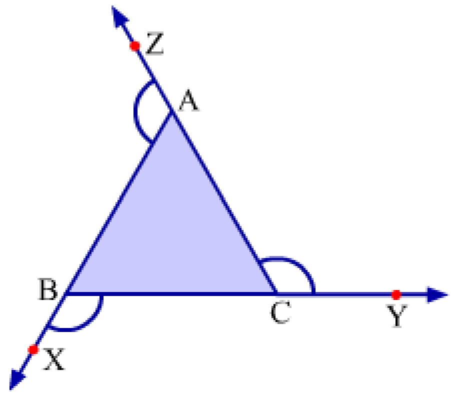
Point O is the centroid of △△ABC and AP, BQ and CR are the medians of sides BC, AC and AB respectively.

Now, let us look at an example.

Example 1:

In the triangle PQR, PS is a median and the length of $\overline{SR}$ = 6.5 cm. Find the length of $\overline{QR}$.



Solution:

Using the exterior angle property, we obtain:

$\angle BAZ = \angle ABC + \angle ACB$ … (1)

$\angle CBX = \angle BAC + \angle ACB$ … (2)

$\angle ACY = \angle BAC + \angle ABC$ … (3)

On adding equations 1, 2 and 3, we obtain:

$\angle BAZ + \angle CBX + \angle ACY = \angle ABC + \angle ACB + \angle BAC + \angle ACB + \angle BAC + \angle ABC$

$\Rightarrow \angle BAZ + \angle CBX + \angle ACY = 2(\angle ABC + \angle ACB + \angle BAC)$

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According to the angle sum property of triangles, we have:

$\angle ABC + \angle ACB + \angle BAC = 180°$

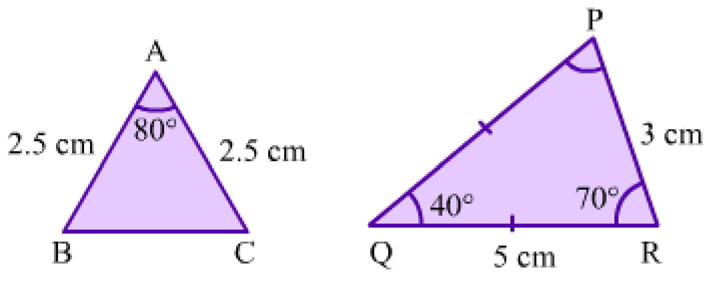
$\therefore \angle BAZ + \angle CBX + \angle ACY = 2 \times 180° = 360°$

Thus, the sum of the three exterior angles is 360°.

Example 2:

Show that AC is the bisector of ∠BAD in the given figure.

Find the missing angles in the following triangles.



(i) (ii)

Solution:

1. In ΔABC, AB = AC = 2.5 cm

   Since the angles opposite equal sides of a triangle are equal, we obtain:

   ∠ABC = ∠ACB

   Let ∠ABC = ∠ACB = x

   By the angle sum property of triangles, we have:

   ∠ABC + ∠ACB + ∠BAC = 180°

   ⇒ $x + x + 80° = 180°$

   ⇒ $2x = 100°$

   ⇒ $x = 50°$

   Thus, ∠ABC = ∠ACB = 50°

2. In ΔPQR, PQ = QR = 5 cm and PR = 3 cm

   Since PQ = QR, we obtain

   ∠QPR = ∠PRQ = 70°

Example 2:

The shown ΔABC is isosceles with AB = AC. Find the measures of ∠BAC, ∠ABC and ∠ACB.

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$PQ = PR$ (Given)

$\therefore \angle PQR = \angle PRQ$ ($\because$ Angles opposite equal sides are equal)

It is given that $\angle QTS = 90°$.

Using the angle sum property in ΔQTS, we obtain:

$\angle TQS + \angle TSQ + \angle QTS = 180°$

$\Rightarrow \angle TQS + \angle TSQ + 90° = 180°$

$\Rightarrow \angle TSQ = 90° - \angle TQS$ … (1)

It is given that $\angle RUS = 90°$.

Using the angle sum property in ΔRUS, we obtain:

$\angle SRU + \angle RUS + \angle RSU = 180°$

$\Rightarrow \angle SRU + 90° + \angle RSU = 180°$

$\Rightarrow \angle RSU = 90° - \angle SRU$ … (2)

$\angle SRU = \angle PRQ$ (Vertically opposite angles)

Also, $\angle PQR = \angle PRQ$

$\therefore \angle SRU = \angle PQR$ … (3)

Now, from equations (2) and (3), we get:

$\angle RSU = 90° - \angle PQR$

$\Rightarrow \angle RSU = 90° - \angle TQS$ … (4) [$\because \angle PQR = \angle TQS$]

From equations (1) and (4), we get:

$\angle TSQ = \angle RSU$

Hence, QS bisects $\angle TSU$.

Now, consider ΔQTS and ΔQVS.

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QS = QS (Common side)

∠TSQ = ∠QSV (∵ ∠TSQ = ∠RSU and ∠RSU = ∠QSV)

ST = SV (Given)

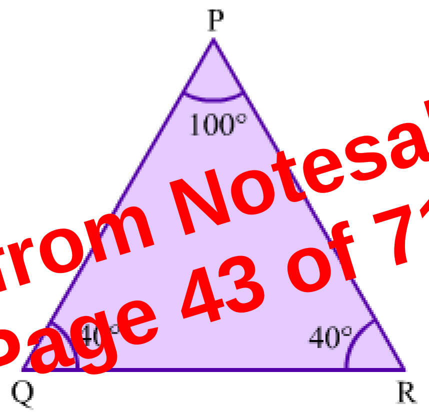
Thus, by the SAS congruency rule, we get:

ΔQTS ≅ ΔQVS

Sides Opposite to Equal Angles of a Triangle are Equal

Observing the Equal Angles and the Sides Opposite to Them in an Isosceles Triangle

Consider the following ΔPQR.



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Is ΔPQR isosceles? We know that if two sides of a triangle are equal (or congruent), then the triangle is isosceles. However, in ΔPQR, two angles are equal (or congruent). We have studied that the angles opposite to equal (or congruent) sides of an isosceles triangle are equal (or congruent). Is the converse of this property also true?

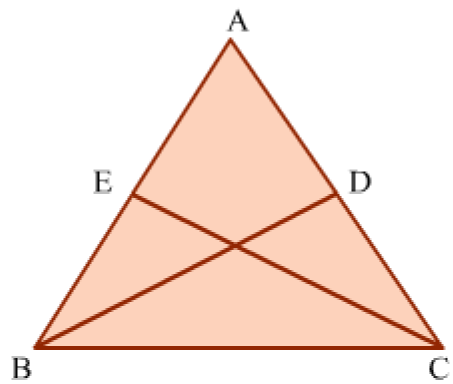
In this lesson, we will study about the equality of sides opposite equal angles in an isosceles triangle. We will also solve some examples related to this concept.

Sides Opposite To Equal Angles of a Triangle Are Equal

Whiz Kid

In an isosceles triangle, the medians drawn from the base vertices to the opposite sides are of equal length**.**

For example:



The shown ΔABC is isosceles such that AB = AC. BD and CE are the respective medians from vertices B and C to sides AC and AB. Therefore, BD = CE.

Solved Examples

Easy

Example 1:

In a ΔABC, ∠BAC = 2x and ∠ABC = ∠ACB = x. Find the value of x and hence show that AB = AC.

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Solution:

It is given that ∠BAC = 2x and ∠ABC = ∠ACB = x.

By applying the angle sum property, we obtain:

∠BAC + ∠ABC + ∠ACB = 180°

⇒ 2x + x + x = 180°

⇒ 4x = 180°

⇒ x = 45°

So, ∠BAC = 2x = 2 × 45° = 90° and ∠ABC = ∠ACB = x = 45°

The given triangle can be drawn as is shown.

AB = AB (Common side)

So, by the SAS congruence rule, we obtain:

ΔABD ≅ ΔABC

⇒ AD = AC and ∠DAB = ∠BAC (By CPCT)

Let ∠BAC be x. Then, ∠DAB will also be x.

Now, ∠DAC = ∠DAB + ∠BAC

⇒ ∠DAC = $x + x$

⇒ ∠DAC = $2x$

⇒ ∠DAC = ∠ACB (∵ ∠ACB = 2∠BAC = $2x$)

⇒ DC = AD (∵ Sides opposite equal angles are equal)

Since BC = DB, we have:

DC = 2BC

⇒ 2BC = AD

⇒ 2BC = AC (∵ We have proved AD = AC)

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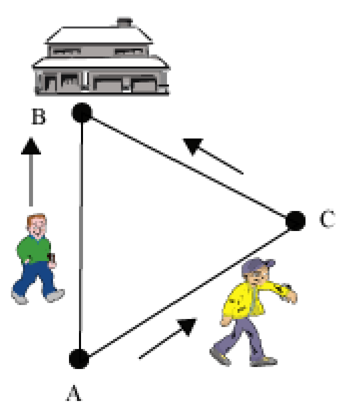
Triangle Inequalities

Rohit and Mohit are brothers. Their house is located at position B, as shown in the following figure.



$\Rightarrow \angle 2 > \angle PRQ$ … (2) [$\because \angle PRQ = \angle SRQ$]

From (1) and (2), we have:

$\angle 1 > \angle PRQ$ ... (3)

Now, $\angle 1$ is a part of $\angle PQR$.

So, $\angle PQR > \angle 1$ … (4)

Thus, from (3) and (4), we can conclude that:

$\angle PQR > \angle PRQ$

Solved Examples

Easy

Example 1:

In the given ΔPQR, PQ is greater than PR. Also, QS and RS are the respective bisectors of $\angle PQR$ and $\angle PRQ$. Prove that $\angle SRQ > \angle SQR$.

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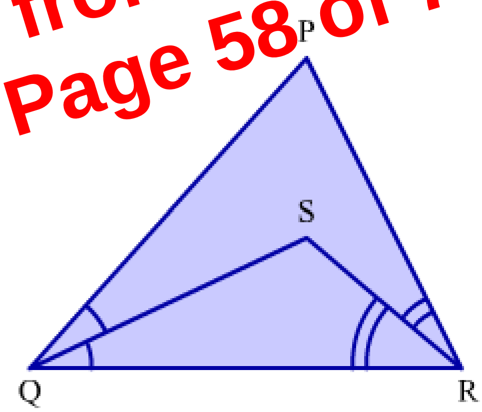

Solution:

In ΔPQR, we have:

PQ > PR (Given)

$\Rightarrow \angle PRQ > \angle PQR$ ($\because$ Angle opposite longer side is greater)

$$\Rightarrow \frac{\angle PRQ}{2} > \frac{\angle PQR}{2}$$

$\Rightarrow \angle ACB > \angle ABC$ ($\because$ Angle opposite longer side is greater)

On adding $\angle 1$ to both sides, we get:

$\angle ACB + \angle 1 > \angle ABC + \angle 1$

$\Rightarrow \angle ACB + \angle 2 > \angle ABC + \angle 1$ … (1) [$\because$ AD bisects $\angle BAC$; $\angle 1 = \angle 2$]

By the exterior angle property, we have:

$\angle ADB = \angle ACB + \angle 2$ … (2)

Similarly, $\angle ADC = \angle ABC + \angle 1$ … (3)

Thus, by using (1), (2) and (3), we can conclude that:

$\angle ADB > \angle ADC$

Observation of the Sides of a Triangle by Seeing the Angles

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Consider the following triangular racetrack where two cars start from different points A and B to reach the finish point C.

The red car has to travel at an angle of 65°, while the blue car has to travel at an angle of 50° with respect to line AB to reach the finish point C. Do you think any one car has an advantage over the other?

On observing the triangular track, it seems that path BC is longer than path AC. So, clearly, the red car has an advantage over the blue car. Also, the angle opposite BC is greater than the angle opposite AC. So, what does this tell us about the relation between the sides and angles of the triangular racetrack?

Let us go through this lesson to learn about the relation between the sides and angles of a triangle. We will also solve some problems based on this relation.

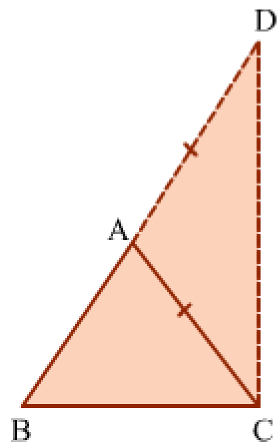
In a Triangle, the Side Opposite the Greater Angle is

Longer

We have studied that in a triangle having two unequal sides, the angle opposite the longer side is greater. The converse of this property is also true. It states that:

If two angles of a triangle are unequal, then the greater angle has the longer side opposite it. In other words, the smaller angle has the shorter side opposite it.

Consider the following ΔPQR.



In ΔACD, we have:

$AD = AC$ … (1)

We know that in an isosceles triangle, the angles opposite equal sides are equal.

$\therefore \angle ACD = \angle ADC$

$\Rightarrow \angle ACD + \angle ACB > \angle ADC$

$\Rightarrow \angle BCD > \angle ADC$

We know that the side opposite the greater angle is longer. So, we obtain, $BD > BC$

$\Rightarrow AB + AD > BC$ ($\because BD = AB + AD$)

$\Rightarrow AB + AC > BC$ (Using equation 1)

$\Rightarrow BC - AC < AB$

Similarly, we can prove that $AB - BC < AC$ and $AC - AB < BC$.

Thus, we have proved that the difference between any two sides of a triangle is less than the third side.

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