culus Review: Key Concepts

Precalculus Review: Key Concepts

Precalculus serves as a foundation for calculus and covers a variety of essential mathematical concepts. Here's a review of the key topics in precalculus:

1. Functions

Definition: A function is a relation where each input has a unique output.

Types of Functions:

Linear:

Quadratic:

Polynomial:

Rational: , where and are polynomials

Exponential:

Logarithmic:

Domain and Range: The domain refers to the set of possible inputs (x-values), and the range refers to the set of possible outputs (y-values).

2. Polynomial Functions

Degree of a Polynomial: The highest exponent of .

Factoring: Methods like factoring by grouping, using the difference of squares, or applying the quadratic formula.

Roots/Zeros: Solutions to , which correspond to the x-intercepts of the graph.

3. Rational Functions

Asymptotes:

Vertical Asymptotes occur where the denominator is zero and the function becomes undefined.

Horizontal Asymptotes describe the behavior of the function as approaches infinity.

Simplifying Rational Expressions: Canceling common factors in the numerator and denominator.

4. Exponential and Logarithmic Functions

Exponential Functions: The general form is , where and .

Growth and Decay: Exponential functions model growth (when) or decay (when).

Logarithmic Functions: The inverse of exponential functions. The general form is , where and .

Logarithmic Properties:

5. Trigonometry

Basic Trigonometric Functions: Sine (), cosine (), tangent (), secant (), cosecant (), and cotangent ().

Unit Circle: The unit circle is crucial for understanding the values of trigonometric functions for all angles.

Trigonometric Identities: Important identities like the Pythagorean identities, angle sum/difference formulas, and double angle formulas.

Graphs of Trigonometric Functions: Understanding the amplitude, period, phase shift, and vertical shift of the graphs of sine, cosine, and other trigonometric functions.

6. Sequences and Series

Arithmetic Sequences: A sequence where each term is the sum of the previous term and a constant. The formula is .

Geometric Sequences: A sequence where each term is multiplied by a constant. The formula is .

Summation: The sum of the terms of a sequence, especially in the case of arithmetic or geometric series.

7. Limits (Introduction to Calculus)

Limit Concept: The value that a function approaches as the input approaches a certain point.

Limit Laws: The properties and rules for calculating limits, including sum, difference, product, quotient, and power laws.

Continuity: A function is continuous at a point if the limit as approaches the point equals the function's value at that point.

8. Conic Sections

Parabolas: The graph of quadratic functions, represented by .

Circles: Equations of the form .

Ellipses: Equations of the form .

Hyperbolas: Equations of the form .

9. Systems of Equations

Solving Linear Systems: Methods include substitution, elimination, and graphing.

Nonlinear Systems: Systems involving quadratics, exponentials, or other non-linear functions.

10. Matrices (Introductory)

Matrix Operations: Addition, subtraction, and multiplication of matrices.

Determinants and Inverses: Methods for solving systems of equations using matrices (e.g., Cramer's rule, inverse matrices).

These concepts provide the groundwork for more advanced study in calculus, and mastering them is essential for success in higher-level mathematics.