$$= 2\frac{1}{2\sqrt{2}} \cdot \frac{3}{2\sqrt{2}} + \frac{1}{3}$$

$$= \frac{3}{4} + \frac{1}{3} = \frac{9+4}{12} = \frac{13}{12}$$

9) $S.T. 2\, tan^{-1}(-3) = \frac{\pi}{2} + tan^{-1}\left(\frac{-4}{3}\right)$

Solution

We have $tan^{-1}(-3) - tan^{-1}\left(\frac{-4}{3}\right)$

$=> tan^{-1}\left(\frac{2\times -3}{1-9}\right) + tan^{-1}\left(\frac{4}{3}\right)$

$=> tan^{-1}\left(\frac{-6}{1-9}\right) + tan^{-1}\left(\frac{4}{3}\right)$

$=> tan^{-1}\left(\frac{3}{4}\right) + cot^{-1}\left(\frac{3}{4}\right) = \frac{\pi}{2}$

$\therefore tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$

10.) S.T $2\ \tan^{-1}(-3) = +\frac{\pi}{2} => \frac{1}{2}\left(\frac{-4}{3}\right)$

Solution

Let $\sin^{-1}\frac{1}{2} = \theta => \frac{1}{2} = \sin\theta$

We have $\tan^{-1}\left[2\cos\left(2\ \sin^{-1}\frac{1}{2}\right)\right]$

$=> tan^{-1}[2.\cos(2\,\theta)]$

$=> tan^{-1}[2.(1-2\ \sin^2\theta)]$

$=> tan^{-1}\left[2\left(1-2\left(\frac{1}{4}\right)\right)\right]$

$=> tan^{-1}\left[2\left(1-\frac{1}{2}\right)\right] =>$

$=> tan^{-1}\left[2\left(\frac{1}{2}\right)\right] = \tan^{-1}(1) = \frac{\pi}{4}$

3 marks questions

1) P.T. $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Solution

L.H.S. $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$=> cos^{-1}\left(\frac{4}{5} \cdot \frac{12}{13} - \sqrt{1-\frac{16}{25}}\ \sqrt{1-\frac{144}{169}}\right)$

$$= \tan^{-1}\left(\frac{\frac{6}{7}+\frac{11}{23}}{1-\frac{6}{7}.\frac{11}{23}}\right) = \tan^{-1}\left(\frac{\frac{138+77}{7.23}}{1-\frac{66}{23}}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

4) P.T. $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, -\frac{1}{\sqrt{2}} \leq x \leq 1$

Solution

Put $x = \cos 2\theta \Rightarrow 2\ \theta \ \cos^{-1} x \Rightarrow \theta = \frac{1}{2}\ \cos^{-1} x$

$$= \tan^{-1}\left(\frac{\sqrt{1+\cos 2\,0} \quad -\sqrt{1-cos\ 2\,\theta}}{-\sqrt{1+cos\ 2\,\theta} + -\sqrt{1-cos\ 2\,\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\ \theta} - \sqrt{2\sin^2\ \theta}}{\sqrt{2\cos^2\ \theta} + \sqrt{2\sin^2\ \theta}}\right)$$

$\therefore -\frac{1}{\sqrt{2}} \leq x \leq 1$

$\Rightarrow -\frac{1}{\sqrt{2}} \leq cos\ 2\,\theta \leq 1$

$\Rightarrow \theta \leq 2\,\theta \leq \frac{3\pi}{4}$

$\Rightarrow 0 \leq \theta \leq \frac{3\pi}{8}$

$\therefore \cos\theta > 0, sin\ \theta > 0$

$$= \tan^{-1}\left(\frac{\cos\ \theta - \sin\theta}{\cos\theta + \sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] = \pi - 0 = \left(\frac{\pi}{4} - \frac{1}{2}\ \cos^{-1} x\right)$$

5) P.T. $\cot^{-1} 7 + \cot^{-1} 8 + \cos^{-1} 18 = \cot^{-1} 3$

Solution

We have $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{7}{1}.\frac{1}{8}}\right) + \tan^{-1}\frac{1}{18}$$

$\therefore \cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right)$

$\left(\text{since } x.y = \frac{1}{7}.\frac{1}{8} < 1\right)$

$$= \tan^{-1}\left(\frac{3}{11}\right) + \tan^{-1}\frac{1}{18}$$

$$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}-\frac{1}{18}}\right)$$