3) Evaluate

i) $\begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$ ii) $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

Answer

i) $\Delta = 0 \quad \because R_1 \cong R_3$

ii) $\Delta = \begin{vmatrix} 6(17) & 6(3) & 6(6) \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$

$= 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 0 \quad \because R_1 \cong R_3$

Two mark Questions:

1) Prove the following properties

i) In case of third order determinant, the value of the determinant is unaltered if the rows are changed into columns and columns into rows

ii) If any two rows (or columns) of a determinant are interchanged, then sign of the determinant changes

iii) If any two rows (or columns) of a determinant are identical then the value of the determinant is zero.

Answer: (For proofs refer text books)

i) Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2) \rightarrow (1)$

3) Find the area of the triangle with vertices at the points in each of the following using determinants

i) $(1,0), (6,0), (4,3)$

ii) $(2,7), (1,1), (10,8)$

iii) $(-2,-3), (3,2), (-1,-8)$

Answer

i) Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{15}{2}$ sq. units

ii) $\Delta = \frac{47}{2}$ sq. units

iii) $\Delta = 15$ sq. units

4) Show that the points $A(a, b+c)$, $B(b, c+a)$, $C(c, a+b)$ are collinear

Answer: $\Delta ABC = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$

Operate: $C_1 \to C_1 + C_2$

$\Delta ABC = \frac{1}{2}\begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix}$

$= \frac{1}{2}(a+b+c)\begin{vmatrix} 0 & b+c & 1 \\ 0 & c+a & 1 \\ 0 & a+b & 1 \end{vmatrix} = 0$

$\therefore$ A, B and C are collinear.