Q14. The sum to 10 terms of the series

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\text{ is}$$

(A) $\frac{56}{111}$ (B) $\frac{55}{111}$

(C) $\frac{59}{111}$ (D) $\frac{58}{111}$

Q15. The combined equation of the two lines $ax+by+c=0$ and $a'x+b'y+c'=0$ can be written as $(ax+by+c)(a'x+b'y+c')=0$.

The equation of the angle bisectors of the lines represented by the equation $2x^2+xy-3y^2=0$ is

(A) $x^2-y^2-10xy=0$ (B) $3x^2+5xy+2y^2=0$

(C) $3x^2+xy-2y^2=0$ (D) $x^2-y^2+10xy=0$

Q16. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}, x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(A) $\alpha^2-\beta^2=4\sqrt{3}$ (B) $\alpha^2+\beta^2=\frac{9}{2}$

(C) $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$ (D) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$

Q17. The value of $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\ldots+\frac{1}{49!2!}+\frac{1}{51!1!}$ is :

(A) $\frac{2^{50}}{51!}$ (B) $\frac{2^{51}}{50!}$

(C) $\frac{2^{51}}{51!}$ (D) $\frac{2^{50}}{50!}$

Q18. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be Q. Then the distance of the plane $3x+2y+z+29=0$ from the point Q is

(A) $\frac{22\sqrt{2}}{7}$ (B) $3\sqrt{14}$

(C) $\frac{24\sqrt{2}}{7}$ (D) $2\sqrt{14}$

Q19. The shortest distance between the lines

$$\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}\text{ and }\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}\text{ is}$$

(A) $5\sqrt{3}$ (B) $6\sqrt{3}$

(C) $7\sqrt{3}$ (D) $4\sqrt{3}$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q1. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is…………

Q2. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{\omega} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to………..

Q3. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is………

Q4. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to………..

Q5. Let $f: R \to R$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to………

Q6. If $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{\ell}(11)^{m/n}$ where $\ell, m, n \in N$, m and n are coprime then $\ell + m + n$ is equal to………

Q7. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and x = 2. Then 12 A is equal to………

Q8. Let $a_1 = 8, a_2, a_3, ..., a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is…….

Q9. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is…….

Q10. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to…….

FIITJEE

Solutions to JEE (Main)-2023

PART – A (PHYSICS)

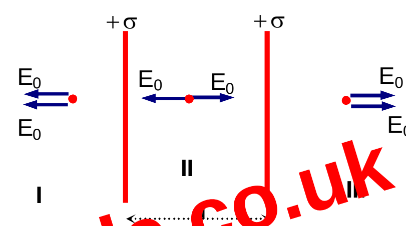
SECTION – A

Sol1. The average kinetic energy of a molecule of the gas $= \frac{3}{2}kT$

$\Rightarrow$The average kinetic energy of a molecule of the gas $\alpha\ T$ (absolute temperature)

Sol2. $E_0 = \frac{\sigma}{2\in_0}$

$\vec{E}_I = -2E_0\hat{n} = -\frac{\sigma}{\in_0}\hat{n}$, $\vec{E}_{II} = \vec{0}$ and $\vec{E}_{III} = 2E_0\hat{n} = \frac{\sigma}{\in_0}\hat{n}$



Sol3. Dimension of b = $= V \equiv \left[M^0L^3T^0\right]$

$$PV^2 = a \equiv \left[\frac{MLT^{-2}}{L^2}\right] \times L^6 = \left[ML^5T^{-2}\right] \Rightarrow \frac{b^2}{a} = \frac{\left[M^0L^6T^0\right]}{\left[ML^5T^{-2}\right]} \equiv \left[M^{-1}LT^2\right]$$

Compressibility $= -\frac{1}{V}\frac{dV}{dP} = \frac{1}{P} = \left[M^{-1}LT^2\right]$

Sol4. Given, $M_P \Rightarrow$ Actual mass of Planet

$M_e = 9M_P \Rightarrow \frac{M_e}{M_P} = 9$

$R_P \Rightarrow$ Actual radius of Planet

$R_e = 2R_P \Rightarrow \frac{R_e}{R_P} = 2$

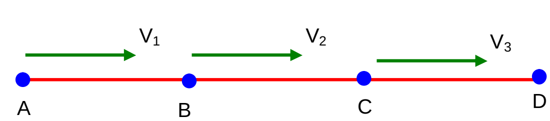
We know escape velocity $= \sqrt{\frac{2GM}{r}}$

$\therefore \frac{v_e}{v_P} = \sqrt{\frac{M_e}{M_P} \times \frac{R_P}{R_e}} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

Hence $v_P = \frac{v_e\sqrt{2}}{3} = \frac{v_e}{3}\sqrt{x} \Rightarrow x = 2$

Sol5. $AB = BC = CD = x$ **and** $S = ut$

$t_{AB} = \frac{x}{v_1}$, $t_{BC} = \frac{x}{v_2}$ **and** $t_{CD} = \frac{x}{v_3}$



$\therefore$ total time $= \frac{3x}{v_{avg}} = t_{AB} + t_{BC} + t_{CD}$

$$= \frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}$$

$$\Rightarrow \frac{3}{v_{avg}} = \frac{v_2v_3 + v_1 + v_2 + v_1v_2}{v_1v_2v_3}$$

$$\therefore v_{avg} = \frac{3v_1v_2v_3}{v_2v_3 + v_1v_2 + v_1v_3}$$

Sol6. Metals have Fermi level inside conduction bond
$\therefore$ D = IV
Intrinsic semiconductor $\rightarrow$ Fermi level b/w valence & conductive bond A = IF
P-typre semiconductor Fermi level near valence hard
C=I

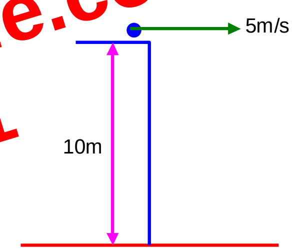
Sol7. 1^{st} Plate $\Rightarrow \frac{I_0}{2}$

2^{nd} plate $\Rightarrow \frac{I_0}{2}\cos^2\phi$

$\therefore$ n^{th} plate $\Rightarrow \frac{I_0}{2}\left(\cos^2\phi\right)^n = \frac{I_0}{2}(2)^n = \frac{I_0}{2^{n+1}} = \frac{I_0}{64}$

$\therefore$ Total no of plates = 6

Sol8. Using conservation of energy



$$\frac{1}{2}mv^2 + mgh = \frac{h}{2} = v_0^2$$

$$\Rightarrow \frac{2s}{2} + 20\times 10 = \frac{1}{2}v_0^2$$

$$\Rightarrow 25 + 200 = v_0^2$$

$$\Rightarrow v_0^2 = 225 \Rightarrow v_0 = 15$$

Sol9. $PV^{\gamma} = \text{constant}$

$TV^{\gamma-1} = \text{constant}$

$TV^{\frac{1}{2}} = \text{constant}$

$T_1V_1^{1/2} = T_2(2V_1)^{1/2}$

$T_2 = \frac{T_1}{\sqrt{2}}$ $\qquad \frac{nR}{r-1}(T_1 - T_2)$

Work done = $\frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{3}{2}-1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{1}{2}} = RT\left(2-\sqrt{2}\right)$

Sol10. $\mu = 7.0\times 10^{-3}\,kg/m$

$T = 70$

$$\therefore v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{7\times 10^{-3}}} = \sqrt{10^4} = 100$$

Sol8. $\frac{dy}{dx} + y\tan x = x\sec x$

I.F.= $e^{\int \tan x\,dx} = e^{\ell n \sec x} = \sec x$

solution $y\sec x = \int x\sec^2 x\,dx$

$y\sec x = x\tan x - \ell n\sec x + c$

$1 = c$

$y\sec x = x\tan x - \ell n\sec x + 1$

$y\left(\frac{\pi}{6}\right) \Rightarrow y\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}\cdot\frac{1}{\sqrt{3}} - \ell n\left(\frac{2}{\sqrt{3}}\right) + 1$

$y = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ell n\left(\frac{2}{e\sqrt{3}}\right)$

Sol9. $\left|\frac{z-2}{z-3}\right| = 2$

for $z = x + iy$

$(x-2)^2 + y^2 = 4\left[(x-3)^2 + y^2\right]$

$3x^2 + 3y^2 - 20x + 32 = 0$

$x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$

centre $(\alpha,\beta) = \left(\frac{10}{3},0\right)$ $\quad r = \sqrt{\frac{100}{9} - \frac{32}{3}} = \frac{2}{3}$

$\alpha = \frac{10}{3}, \beta = 0, \gamma = \frac{2}{3}$

$3(\alpha + \beta + \gamma) = 12$

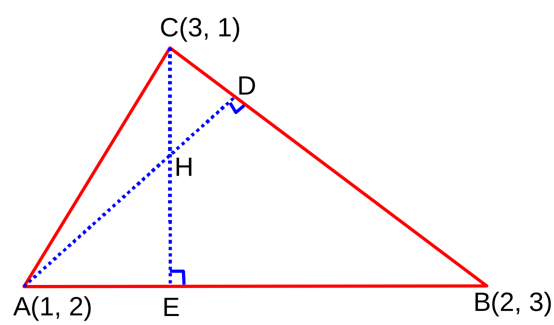
Sol10. Equation AD $y - 2 = \frac{1}{2}(x-1)$ ………(i)

Equation CE $y - 1 = -1(x-3)$ ………(ii)

Solving (i) & (ii) $H(\alpha,\beta) \equiv \left(\frac{5}{3},\frac{7}{3}\right)$

$\alpha + 4\beta = 11$ & $4\alpha + \beta = 9$

$\therefore$ the required quadratic equation is $x^2 - 20x + 99 = 0$



Sol11. $f(x) = 2x + \tan^{-1}x$

$g(x) = \ell n\left(\sqrt{1+x^2} + x\right)$

Let $h(x) = 2x + \tan^{-1}x - \ell n\left(\sqrt{1+x^2} + x\right)$

$h'(x) = 2 + \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} > 0\ \forall x > 0$

$h''(x) = -\frac{2x}{\left(1+x^2\right)^2} + \frac{2x}{\left(1+x^2\right)^{3/2}}$

$$\Sigma t_n = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$$

Sol15. $2x^2 + xy - 3y^2 = 0$ ……..(i)

Equation of angle bisectors between the lines of (i) is $\frac{x^2 - y^2}{2+3} = \frac{xy}{\frac{1}{2}}$

$\Rightarrow x^2 - y^2 = 10xy$

Sol16. $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$

$C_1 + C_2 + C_3$

$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$

$R_3 - R_1, R_2 - R_1$

$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$f(x) = 2 + \sin 2x$

$\alpha = 2 + 1 = 3$ for $x = \frac{\pi}{4}$.

$\beta = 2 + \frac{\sqrt{3}}{2}$ for $x = \frac{\pi}{3}$

$\therefore \beta^2 - 2\sqrt{\alpha} = 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3} = \frac{19}{4}$

Sol17. $\frac{1}{1!\,50!} + \frac{1}{3!\,48!} + \ldots\ldots + \frac{1}{51!\,1!}$

$= \frac{1}{51!}\left[\frac{51!}{1!\,50!} + \frac{51!}{3!\,48!} + \ldots\ldots. + \frac{51!}{51!\,1!}\right]$

$= \frac{1}{51!}\left[{}^{51}C_1 + {}^{51}C_3 + \ldots\ldots + {}^{51}C_{51}\right]$

$= \frac{2^{50}}{(51)!}$

Sol18. Image of $P(2, -1, 3)$ in $x + 2y - z = 0$ be $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{-1} = \frac{-2(\not{2} - \not{2} - 3)}{1+4+1} = 1$

$x = 3, y = 1, z = 2$

$\therefore Q(3, 1, 2)$

$\therefore$ distance of $3x + 2y + z + 29 = 0$ from $Q = \left|\frac{9+2+2+29}{\sqrt{9+4+1}}\right| = \frac{42}{\sqrt{14}} = 3\sqrt{14}$