T-Ratios of an Angle 3θ in terms of θ

- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
- $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^2\theta}{1 - 3\tan^2\theta}$

Hamaray Sath(8) Yaar

- **Y_1 :** $\sin\theta\sin(60^\circ-\theta)\sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$
- **Y_2 :** $\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$
- **Y_3 :** $\tan\theta\tan(60^\circ-\theta)\tan(60^\circ+\theta) = \tan 3\theta$
- **Y_4 :** $\cot\theta\cot(60^\circ-\theta)\cot(60^\circ+\theta) = \cot 3\theta$
- **Y_5 :** $\tan\theta + \tan(60^\circ+\theta) + \tan(120^\circ+\theta) = 3\tan 3\theta$
- **Y_6 :** $\cot\theta + \cot(60^\circ+\theta) + \cot(120^\circ+\theta) = 3\cot 3\theta$
- **Y_7 :** $\cot\theta + \tan\theta = 2\operatorname{cosec} 2\theta$
- **Y_8 :** $\cot\theta - \tan\theta = 2\cot 2\theta$

Golden Point....

- $a\sin x + b\cos x = \sqrt{a^2+b^2}\left(\dfrac{a\sin x}{\sqrt{a^2+b^2}} + \dfrac{b\cos x}{\sqrt{a^2+b^2}}\right)$

  $= \sqrt{a^2+b^2}\,(\sin\alpha\cdot\sin x + \cos\alpha\cos x)$
- $a\sin x + b\cos x = \sqrt{a^2+b^2}\cos(x-\alpha)$
- $a\sin x + b\cos x \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$
- $a\sin x + b\cos x\big|_{MAX} = \sqrt{a^2+b^2}$
- $a\sin x + b\cos x\big|_{MIN} = -\sqrt{a^2+b^2}$

Meaning of Σ and Π

(Σ means summation)

- $\sum_{r=1}^{n}\cos r\theta = \cos\theta + \cos 2\theta + \cos 3\theta + ... + \cos n\theta$

(Π means product)

- $\prod_{r=1}^{n}\sin(r\theta) = \sin\theta\cdot\sin 2\theta\cdot\sin 3\theta....\sin n\theta$

Golden Point

- $\prod_{r=1}^{n}\cos(2^{r-1}\theta)$

  $= \cos\theta\cos 2\theta\cos 2^2\theta\cos 2^3\theta....\cos 2^{n+1}\theta = \dfrac{\sin 2\theta}{2^n\sin\theta}$

Value at some Golden Angles....

- $\sin 18^\circ = \sin\frac{\pi}{10} = \cos 72^\circ = \cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$
- $\cos 36^\circ = \cos\frac{\pi}{5} = \sin 54^\circ = \sin\frac{3\pi}{10} = \frac{\sqrt{5}+1}{4}$
- $\sin 15^\circ = \cos 75^\circ = \sin\frac{\pi}{12} = \cos\frac{5\pi}{12}$

  $= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$
- $\cos 15^\circ = \sin 75^\circ = \cos\frac{\pi}{12} = \sin\frac{5\pi}{12}$

  $= \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$
- $\tan 15^\circ = \tan\frac{\pi}{12} = \cot 75^\circ = \cot\frac{5\pi}{12} = 2-\sqrt{3}$
- $\cot 15^\circ = \cot\frac{\pi}{12} = \tan 75^\circ = \tan\frac{5\pi}{12} = 2+\sqrt{3}$
- $\tan 22.5^\circ = \tan\frac{\pi}{8} = \cot 67.5^\circ = \cot\frac{3\pi}{8} = \sqrt{2}-1$
- $\cot 22.5^\circ = \cot\frac{\pi}{8} = \tan 67.5^\circ = \tan\frac{3\pi}{8} = \sqrt{2}+1$

- $\sin\theta = 0,\ \theta \in n\pi$
- $\operatorname{cosec}\theta$ is ND, $\theta \in n\pi$
- $\cot\theta$ is ND, $\theta \in n\pi$
- $\tan\theta = 0,\ \theta \in n\pi$
- $\cos\theta = 1,\ \theta \in (\text{Even }\pi)$, $\cos\theta = -1,\ \theta \in (\text{Odd }\pi)$
- $\sec\theta = 1,\ \theta \in (\text{Even }\pi)$, $\cos\theta = -1,\ \theta \in (\text{Odd }\pi)$
- $\sin\theta = 1,\ \theta \in \frac{(4n+1)\pi}{2}$
- $\sin\theta = -1,\ \theta \in \frac{(4n-1)\pi}{2}$
- $\operatorname{cosec}\theta = 1,\ \theta \in \frac{(4n+1)\pi}{2}$
- $\operatorname{cosec}\theta = -1,\ \theta \in \frac{(4n-1)\pi}{2}$
- $\cos\theta = 0,\ \theta \in \frac{(2n+1)\pi}{2}$
- $\cot\theta = 0,\ \theta \in \frac{(2n+1)\pi}{2}$
- $\sec\theta, \tan\theta = \text{ND},\ \theta \in \frac{(2n+1)\pi}{2}$