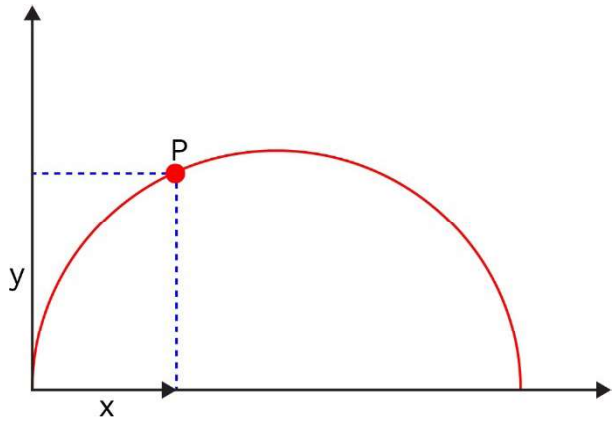
- **EQUATION OF PATH:-**



At time t particle is at point P.

For x-axis:

$$x = u_x t + \frac{1}{2}a_x t^2 \quad \left[\because a_x = 0\right]$$

$$x = u\cos\theta t \Rightarrow t = \frac{x}{u\cos\theta} \quad ...(i)$$

For y-axis:

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$y = u\sin\theta t - \frac{1}{2}gt^2 \quad ...(ii)$$

From equation (i) and (ii)

$$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$\Rightarrow$ Equation of trajectory.

Let $\tan\theta = A$ and $\frac{g}{2u^2\cos^2\theta} = B$

So, $y = Ax - Bx^2$

Concept Reminder:

Equation of Path:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

KEY POINTS

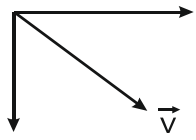
*Equation of trajectory
* Point of projectile

Rack your Brain

The equation of projectile is $y = 16x - \frac{5x^2}{4}$.

Find horizontal range ?

(iii) At P.O.I. (Point of Impact)



$$\vec{v}_{P.O.I.} = u\cos\theta\hat{i} - u\sin\theta\hat{j}$$

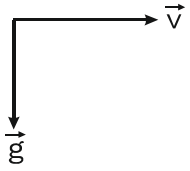
(iv) At any point for y-axis

$$\vec{v}_y = \vec{u}_y + \vec{a}_y t$$

Throughout the motion $F_x = 0, a_x = 0$,

so horizontal component of velocity $(u\cos\theta = \text{constant})$

At highest point acceleration and velocity are mutually perpendicular to reach other.



Concept Reminder:

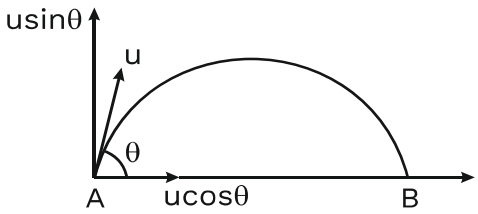
At highest point of projectile
$a \perp v$

Rack your Brain

The velocity of a projectile at point of projection is $2i + 3\hat{j}$. Find its velocity of point of impact.

Time of Flight:

The time taken by projectile between point of projection and point of impact is known as time of flight.



When projectile reach at point of impact then its height becomes zero.

Definitions

The time taken by projectile between point of projection and point of impact is known as time of flight.

If time taken is T during motion between points A to B

So, $\vec{S}_y = \vec{u}_y T + \frac{1}{2}\vec{a}_y T^2$

$$0 = \vec{u}_y T - \frac{\vec{g}}{2}T^2$$

$$T = \frac{2u_y}{g}$$

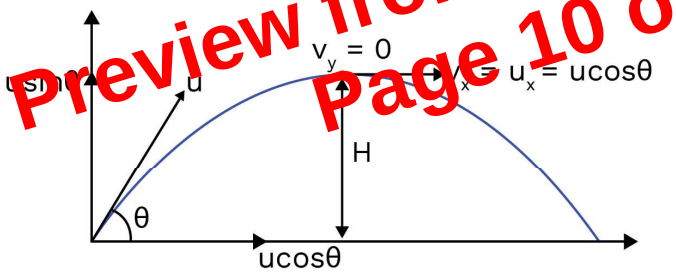
$$T = \frac{2u\sin\theta}{g}$$

Definitions

During projectile motion maximum height attained by object on vertical axis is known as maximum height.

Maximum Height :-

During projectile motion maximum height attained by object on vertical axis is known as maximum height.



Rack your Brain

For a projectile, what is the ratio of maximum height reached to the square of flight time?

At maximum height

$$v_y = 0$$

so, $v_y^2 = u_y^2 + 2a_y S_y$

$$0 = u_y^2 - 2gH$$

$$H = \frac{u_y^2}{2g}$$

$$H = \frac{u^2\sin^2\theta}{2g}$$

at $\theta = 90°$

H becomes maximum, so greatest height

Q2 **A body of mass m is thrown at angle θ with horizontal from ground. The equation of trajectory is $y = \sqrt{3}x - \frac{gx^2}{2}$. Find-**

1. Angle of projection
2. Initial velocity
3. Time of flight
4. Maximum height
5. Range
6. Horizontal and vertical component of initial velocity.

A2 Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = \sqrt{3}x - \frac{gx^2}{2}$$

1. $\tan\theta = \sqrt{3}$

$\theta = 60°$

2. $u^2\cos^2\theta = 1$

$u^2 \times \cos^2 60° = 1$

$u^2 \times \frac{1}{4} = 1$

$u = 2\,m/s$

3. Time of flight $T = \frac{2u\sin\theta}{g}$

$T = \frac{2\times 2\times \sin 60°}{10} = 0.2\sqrt{3}$ sec.

4. $H = \frac{u^2\sin^2\theta}{2g} = \frac{4\times 3}{4\times 2\times 10} = 0.15$ m

5. $y = \sqrt{3}x - \frac{gx^2}{2} = \sqrt{3}\left[1 - \frac{x}{\frac{2\sqrt{3}}{g}}\right]$

$R = \frac{2\sqrt{3}}{g} = 0.2\sqrt{3}\,m$

Q9 **Body of mass m is thrown at an angle θ with horizonal from ground horizontal range is 200 m and time of flight is 10 sec. Find-**
1. Maximum height **2. Initial speed**

A9 Time of flight =

$$T = \frac{2u\sin\theta}{g} = 10$$

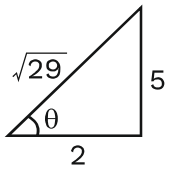
$$u\sin\theta = \frac{10\times 10}{2} = 50 \text{ m/s}$$

$$\text{Range}(R) = 200 = \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$200 = \frac{2(u\sin\theta)^2\times\cos\theta}{g\sin\theta}$$

$$\tan\theta = \frac{2\times(50)^2}{10\times 200} = \frac{5000}{2000} = \frac{5}{2}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{(50)^2}{2\times 10} = 125\,\text{m}$$



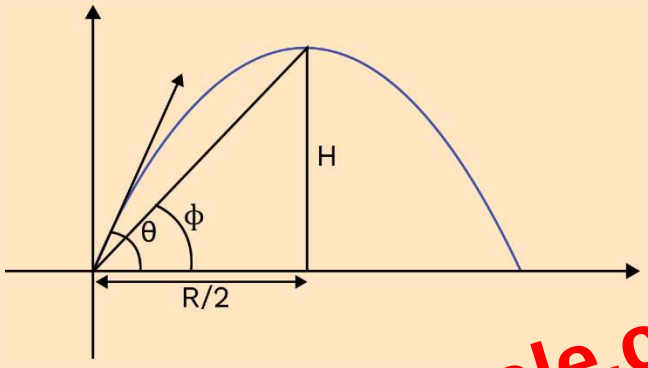
$$\sin\theta = \frac{5}{\sqrt{29}}$$

$$T = \frac{2u\sin\theta}{g}$$

$$u = \frac{g\cdot T}{2\sin\theta}$$

$$u = \frac{10\times 10}{2\times 5}\sqrt{29} = 10\sqrt{29}$$

Q12 **Projectile of mass m is thrown with initial velocity u at an angle θ with horizontal from ground. Find angle made by straight line which join topmost point according P.O.P with horizontal.**



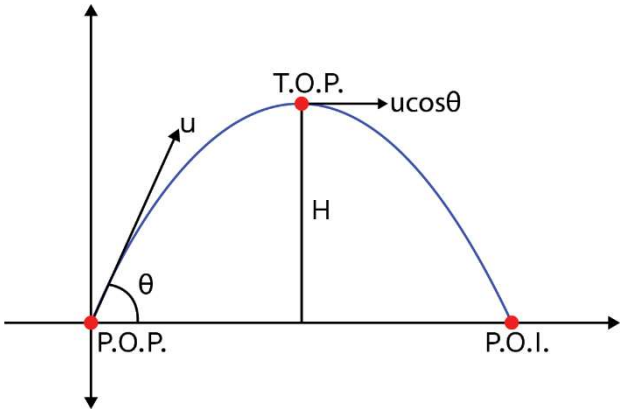
A12

$$\tan\phi = \frac{2H}{R}$$

$$\tan\phi = \frac{2\times u^2 \sin^2\theta \cdot g}{2g \times 2u^2 \sin\theta\cos\theta}$$

$$\tan\phi = \frac{\tan\theta}{2}$$

Kinetic energy, gravitational, potential energy and total mechanical energy when projectile thrown from grown:-

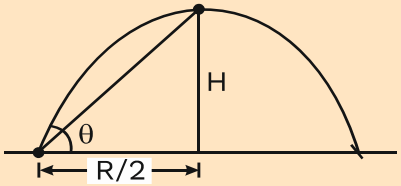


(i) $G.P.E = mgh$

(ii) $KE = \frac{1}{2}mu^2$

Q15 **If a projectile of mass m is thrown at an angle θ with horizontal from ground then average velocity in between**

1. P.O.P and T.O.P **2. P.O.P and P.O.I**



A15 **1.** $\Delta v = \dfrac{\text{Total displacement}}{\text{Total time}}$

$$\Delta v = \frac{\sqrt{\left(\frac{R}{2}\right)^2 + H^2}}{T}$$

$$\Delta v = \frac{\sqrt{\left(\frac{2u^2 \sin\theta\cos\theta}{2\times g}\right)^2 + \left(\frac{u^2\sin^2\theta}{2g}\right)^2}}{\frac{u\sin\theta}{g}}$$

$$\Delta v = \frac{\sqrt{\frac{u^4\sin^2\theta}{4g^2}\left[\left(4\cos^2\theta\right) + \left(\sin^2\theta\right)\right]}}{\frac{u\sin\theta}{g}}$$

$$\Delta v = \frac{\frac{u^2\sin\theta}{2g}}{\frac{u\sin\theta}{g}}\sqrt{3\cos^2\theta + 1}$$

$$\Delta v = \frac{u}{2}\sqrt{3\cos^2\theta + 1}$$

2. $\Delta v = \dfrac{R}{T}$

$$\Delta v = \frac{2u^2\sin\theta\cos\theta\times g}{g\times 2u\sin\theta}$$

$$\Delta v = u\cos\theta = \left(v_{avg}\right)$$