4. Forces and Laws of Motion

Introduction

This chapter delves into the consequences of **motion**, focusing on the concept of **force**, which is defined as a **push** or **pull** experienced by a body or system. The study of motion is heavily influenced by the choice of **reference frame.**

Force

> Force is any influence that causes an object to undergo a certain change, which may be with respect to its movement, direction, or geometrical construction.

In simpler terms, force can:

- Change an object's **velocity** (accelerate or decelerate).
- **Deform** a flexible object.

Force is a **vector quantity**, possessing both **magnitude** and **direction**. It is measured in **Newtons (N)**. When multiple forces act on a body, the **net force** is the **vector addition** of all forces.

Free Body Diagram (FBD)

A **free body diagram (FBD)** is a representation of all the **external forces** acting on an object.

Weight

> The **weight** of an object is the force with which the earth attracts that object towards its center.

Illustration 5: Two forces

$$F_1$$

and

$$F_2$$

act on a 2 kg mass. If

$$F_1 = 10N$$

and

$$F_2 = 5N$$

, find the acceleration.

Solution: Apply Newton's second law of motion. Acceleration will be in the direction of the net force.

$$F = \sqrt{10^2 + 5^2 + 2*10*5*cos(120)} = \sqrt{100 + 25 - 50} = \sqrt{75} = 5\sqrt{3}N$$

$$a = \frac{F}{m} = \frac{5\sqrt{3}}{2} = 2.5\sqrt{3}m/sec^2$$

$$\tan(\theta) = \frac{5\sin(120)}{10 + 5\cos(120)} = \frac{5\frac{\sqrt{3}}{2}}{10 + 5(-\frac{1}{2})} = \frac{5\sqrt{3}}{2} * \frac{2}{15} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\theta = 30$$

Therefore acceleration is

$$2.5\sqrt{3}m/sec^2$$

at an angle

$$30^\circ$$

with the direction of

$$F_1$$

.

(c) Third Law: For every action, there is an equal and opposite reaction.

Masterjee Concepts: Working with Laws of Motion

1. **Decide the system**: Identify the object or combination of objects to apply the laws of motion to.
2. **Identify the forces**: List all forces acting on the system due to external objects.
3. **Make a Free Body Diagram (FBD)**: Represent the system as a point and draw force vectors.
4. **Choose the axes and Write Equations**: Select mutually perpendicular X-Y-Z axes.

- If forces are coplanar, use X and Y axes.
- If the system is in equilibrium, any mutually perpendicular directions can be chosen.

Write equations by equating the sum of force components to the product of mass and acceleration.

Note:

- If the system is in equilibrium:

$$\Sigma F_x = 0$$

and

$$\Sigma F_y = 0$$

- If the system is collinear, the second equation is not needed.

Impulse

The impulse of a force is defined as the product of the average force

$$F$$

and the time interval

$$\Delta t$$

during which the force acts.

Solving the equations,

$$a = \frac{F_1 - F_2}{M_1 + M_2}$$

$$N = \frac{M_2 F_1 + M_1 F_2}{M_1 + M_2}$$

Illustration 11: A rope of length L is pulled by a constant force F. What is the tension in the rope at a distance x from one end where the force is applied?

Solution:

$$massperunitlength = \frac{M}{L}$$

$$F - T = (\frac{M}{L} * (L - x)) * a$$

$$F = Ma$$

$$a = \frac{F}{M}$$

$$F - T = \frac{M}{L} * (L - x) * \frac{F}{M}$$

$$F - T = \frac{L - x}{L} * F$$

$$T = F - \frac{L - x}{L} * F = F * (1 - \frac{L - x}{L}) = F * (\frac{L - L + x}{L}) = F * \frac{x}{L}$$

$$T = F * \frac{x}{L}$$

Illustration 12: Two blocks each having mass of 20 kg rest on frictionless surfaces as are shown in the Figure 4.30. Assume that the pulleys to be light and frictionless. Now, find: (a) The time required for the block A to move 1 m down the plane, starting from rest; (b) The tension in the cord connecting the blocks.

$$sin(\theta) = 3/5$$

Solution:

Applying Newtons laws

$f = mr\omega^2$

From a Non-Inertial Frame (Fixed on the Table)

- Normal reaction (N) balances weight (mg)
- Centrifugal force ($mr\omega^2$) acts radially outward
- Friction (f) balances the centrifugal force

$f = mr\omega^2$

Illustration: Simple Pendulum in Vertical Circle

A bob of mass m is attached to a string of length L and oscillates in a vertical circle. Its speed is v when the string makes an angle θ with the vertical.

Tension in the String

Applying Newton's Second Law

$T - mg\cos(\theta) = \frac{mv^2}{L}$ Therefore, $T = mg\cos(\theta) + \frac{mv^2}{L}$

Magnitude of Net Force

The net force is:

$F_{net} = \sqrt{(mg\sin(\theta))^2 + (\frac{mv^2}{L})^2} = m\sqrt{g^2\sin^2(\theta) + \frac{v^4}{L^2}}$

Illustration: Ball in Rotating Hemispherical Bowl

A hemispherical bowl of radius R rotates about its vertical axis. A small ball inside rotates with the bowl without slipping.

Angular Speed of the Bowl

To solve for the angular speed w, we must use the formula:

- $F = \frac{dp}{dt} = \frac{d(mv)}{dt}$

Tension

- $F = F_{net} = ma$ or $a = \frac{F_{net}}{m}$
- $F = ma$
- $F = 0$
- $T \sin\theta = ma_x$
- $F = 0$
- $T \cos\theta = mgy$
- $mg = \tan^{-1}(\frac{a_x}{g})$

Impulse

- Impulse = change in momentum
- $Ft = mv - mv_0$

Friction

- $f_{max} = \mu_s R$ (limiting friction)
- $f_k = \mu_k R$ (kinetic friction)
    - μ_s = coefficient of static friction
    - μ_k = coefficient of kinetic friction
- Angle of Friction: $\tan\lambda = \frac{f_{max}}{R} = \mu$ or $\lambda = \tan^{-1}(\mu)$

Pseudo Force

- $F = -ma$; where m = mass of the object, a = acceleration of the reference frame

Circular Motion

- Linear velocity v and Angular velocity ω are related by $v = R\omega$ (R = radius of circular path)