

(iii) If $f(x)$ is increasing function on (a, b) , then tangent at every point on the curve $y = f(x)$ makes an acute angle θ with the positive direction of x -axis.

$$\tan \theta > 0 \Rightarrow \frac{dy}{dx} > 0 \text{ or } f'(x) > 0 \text{ for all } x \in (a, b).$$

(iv) Let f be a differentiable real function defined on an open interval (a, b) .

- If $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is increasing on (a, b) .
- If $f'(x) < 0$ for all $x \in (a, b)$, then $f(x)$ is decreasing on (a, b) .

(v) Let f be a function defined on (a, b) .

- If $f'(x) > 0$ for all $x \in (a, b)$ except for a finite number of points, where $f'(x) = 0$, then $f(x)$ is increasing on (a, b) .
- If $f'(x) < 0$ for all $x \in (a, b)$ except for a finite number of points, where $f'(x) = 0$, then $f(x)$ is decreasing on (a, b) .

Properties of Monotonic Functions

1. If $f(x)$ is strictly increasing function on an interval $[a, b]$, then f^{-1} exist and also a strictly increasing function.
2. If $f(x)$ is strictly increasing function on $[a, b]$ such that it is continuous, then f^{-1} is continuous on $[f(a), f(b)]$.
3. If $f(x)$ and $g(x)$ are strictly increasing (or decreasing) function on $[a, b]$, then $g \circ f(x)$ is strictly increasing (or decreasing) function on $[a, b]$.
4. If one of the two functions $f(x)$ and $g(x)$ is strictly increasing and other a strictly decreasing, then $g \circ f(x)$ is strictly decreasing on $[a, b]$.
5. If $f(x)$ is continuous on $[a, b]$, such that $f'(c) \geq 0$ ($f'(c) > 0$) for each $c \in (a, b)$ is strictly increasing function on $[a, b]$.
6. If $f(x)$ is continuous on $[a, b]$ such that $f'(c) \leq 0$ ($f'(c) < 0$) for each $c \in (a, b)$, then $f(x)$ is strictly decreasing function on $[a, b]$.

Maxima and Minima of Functions

1. A function $y = f(x)$ is said to have a local maximum at a point $x = a$. If $f(x) \leq f(a)$ for all $x \in (a - h, a + h)$, where h is somewhat small but positive quantity.

In order to find the global maximum and minimum of $f(x)$ in $[a, b]$, find out all critical points of $f(x)$ in $[a, b]$ (i.e., all points at which $f'(x) = 0$) and let $f(c_1), f(c_2), \dots, f(c_n)$ be the values of the function at these points.

Then, $M_1 \rightarrow$ Global maxima or greatest value.

and $M_1 \rightarrow$ Global minima or least value.

where $M_1 = \max \{ f(a), f(c_1), f(c_2), \dots, f(c_n), f(b) \}$

and $M_1 = \min \{ f(a), f(c_1), f(c_2), \dots, f(c_n), f(b) \}$

Then, M_1 is the greatest value or global maxima in $[a, b]$ and M_1 is the least value or global minima in $[a, b]$.

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