T, and u is an associated eigenvector.

(4) Suppose $T \in L(F^2)$ is defined by $T(w,z) = (-z,w)$. Find the minimal polynomial of T.

Observe T's action on (w,z): $T(w,z) = (-z,w) \Rightarrow T^2(w,z)$. Then, $T^2(w,z) = T(-z,w) = (-w,-z)$. This yields that $T^2(w,z) = -I(w,z)$, where I is the identity of F^2. Hence, $T^2 + I = 0$, implying that the minimal polynomial $m(T)$ is $m(t) = t^2 + 1$.

(5) Suppose $L \in L(V)$ and $p \in P(F)$. Prove that there exists a unique $r \in P(F)$ such that $p(T) = r(T)$ and deg(r) is less than the degree of the minimal polynomial of T.

Let m_L be the minimal polynomial L with $\deg(m_L) = n$. Using polynomial division, we find that $p(t) = m_L(t)q(t) + r(t)$ with $\deg(r) < n$. Then, when we apply L, we get $p(L) = m_L(L)q(L) + r(L)$. However, since $m_L(L) = 0$, $p(L) = r(L)$. Given anything produced by the division algorithm is used, there exists a single $r(t)$ for each $p(t)$ given an $m_L(t)$.

(6) Suppose V is finite-dimensional and $T \in L(V)$ has a minimal polynomial $4 + 5z - 6z^2 - 7z^3 + 2z^4 + z^5$. Find the minimal polynomial of T^{-1}.

From the minimal polynomial $m_T(z)$, we know that

$$(m_T)(T) = 4I + 5T - 6T^2 - 7T^3 + 2T^4 + T^5 = 0$$

Multiplying the entire equation by T^{-5}, we get

$$4T^{-5} + 5T^{-4} - 6T^{-3} - 7T^{-2} + 2T^{-1} + I = 0$$

Substitute $w = T^{-1}$ into the equation:

$$4w^5 + 5w^4 - 6w^3 - 7w^2 + 2w + 1 = 0$$

Then, we reorder the polynomial into standard form, giving

$$w^5 + \frac{5}{4}w^4 - \frac{3}{2}w^3 - \frac{7}{4}w^2 + \frac{1}{2}w + \frac{1}{4} = 0$$

Therefore, our minimal polynomial of T^{-1} is

$$z^5 + \frac{5}{4}z^4 - \frac{3}{2}z^3 - \frac{7}{4}z^2 + \frac{1}{2}z + \frac{1}{4}$$