4.2 Proofs Involving Congruence of Integers

Background Knowledge

An integer is even if it's in the form 2q and odd if it's 2q + 1. Two integers have the same parity (both even or odd) if 2 divides their difference.

Novel Knowledge

Integers can be expressed as 3q, 3q + 1, or 3q + 2 based on their remainder when divided by 3. If two integers have the same remainder when divided by 3, then 3 divides their difference.

The concept of congruence is when one integer is similar to another modulo a third. For instance, 5 and 7 have the same remainder when divided by 4.

Modulo Notation

Every integer can be represented in forms related to division by 2 or 3. Depending on its form, integer will be congruent to 0, 1, or 2 modulo 3, represented as $x \equiv 0 \pmod 3$, $x \equiv 1 \pmod 3$, or $x \equiv 2 \pmod 3$.

When divided by 4, every integer x will be congruent to 0, 1, 2, or 3 modulo 4. This is represented as $x \equiv 0 \pmod 4$, $x \equiv 1 \pmod 4$, $x \equiv 2 \pmod 4$, and $x \equiv 3 \pmod 4$.

This pattern of congruence can be extended for divisions by any integer n that's 5 or greater.