

Maths Assignment

12. Test the convergence of the series.

$$\frac{1}{1 \cdot 2 \cdot 3} - \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} - \dots$$

$$\sum (-1)^{n+1} u_n = \sum (-1)^{n+1} \frac{1}{n(n+1)(n+2)}$$

$$u_n = \frac{1}{n(n+1)(n+2)}$$

$$u_{n+1} = \frac{1}{(n+1)(n+2)(n+3)}$$

To prove: $u_n > u_{n+1}$

$$u_n - u_{n+1} > 0$$

$$= \frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}$$

$$= \frac{n+3 - n}{n(n+1)(n+2)(n+3)}$$

$$= \frac{3}{n(n+1)(n+2)(n+3)} > 0$$

$$\therefore u_n - u_{n+1} > 0$$

$\therefore$, u_n is a monotonic decreasing sequence.

$$\lim_{n \rightarrow \infty} u_n = \lim_{n \rightarrow \infty} \frac{1}{n(n+1)(n+2)}$$

$$= \frac{1}{\infty(\infty+1)(\infty+2)} = \frac{1}{\infty}$$

$$\lim_{n \rightarrow \infty} u_n = 0$$

$\therefore$ The given series is convergent.