C-14. Prove that :

(i) $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

(ii) $\dfrac{2\sin\theta \quad \tan\theta \quad (1-\tan\theta) + 2\sin\theta \quad \sec^2\theta}{(1+\tan\theta)^2} = \dfrac{2\sin\theta}{(1+\tan\theta)}$

(iii) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \pm(\sec A - \tan A)$

(iv) $\dfrac{\cos A \operatorname{cosec} A - \sin A \sec A}{\cos A + \sin A} = \operatorname{cosec} A - \sec A$

(v) $\dfrac{1}{\sec\alpha - \tan\alpha} - \dfrac{1}{\cos\alpha} = \dfrac{1}{\cos\alpha} - \dfrac{1}{\sec\alpha + \tan\alpha}$

(vi) $\dfrac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \dfrac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

Section (D) : Conditional Identities & Trigonometric Series

D-1. For all values of α, β, γ prove that,

$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma) = 4\cos\dfrac{\alpha+\beta}{2} \cdot \cos\dfrac{\beta+\gamma}{2} \cdot \cos\dfrac{\gamma+\alpha}{2}$.

D-2. If $x + y + z = \dfrac{\pi}{2}$ show that, $\sin 2x + \sin 2y + \sin 2z = 4\cos x \cos y \cos z$.

D-3. If $x + y = \pi + z$, then prove that $\sin^2 x + \sin^2 y - \sin^2 z = 2\sin x \sin y \cos z$.

D-4. If $A + B + C = 2S$ then prove that

$\cos(S - A) + \cos(S - B) + \cos(S - C) + \cos S = 4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

D-5. If $A + B + C = 0°$ then prove that $\sin 2A + \sin 2B + \sin 2C = -4\sin A \sin B \sin C$

D-6. If ϕ is the exterior angle of a regular polygon of n sides and θ is any constant, then prove that $\sin\theta + \sin(\theta + \phi) + \sin(\theta + 2\phi) +$ ..... up to n terms $= 0$

D-7. Prove that $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \ldots\ldots + \sin^2 n\theta = \dfrac{n}{2} - \dfrac{\sin n\theta \cos(n+1)\theta}{2\sin\theta}$

D-8. Prove that :

(i) $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7} = \dfrac{1}{8}$

(ii) $\cos\dfrac{\pi}{11}\cos\dfrac{2\pi}{11}\cos\dfrac{3\pi}{11}\cos\dfrac{4\pi}{11}\cos\dfrac{5\pi}{11} = \dfrac{1}{32}$

Section (E) : Range of Trigonometric Expressions

E-1. Find the extreme values of $\cos x \cos\left(\dfrac{2\pi}{3} + x\right)\cos\left(\dfrac{2\pi}{3} - x\right)$

E-2. Find the maximum and minimum values of following trigonometric functions

(i) $\cos 2x + \cos^2 x$ (ii) $\cos^2\left(\dfrac{\pi}{4} + x\right) + (\sin x - \cos x)^2$

E-3. Find the greatest and least value of y

(i) $y = 10\cos^2 x - 6\sin x \cos x + 2\sin^2 x$

(ii) $y = 3\cos\left(\theta + \dfrac{\pi}{3}\right) + 5\cos\theta + 3$