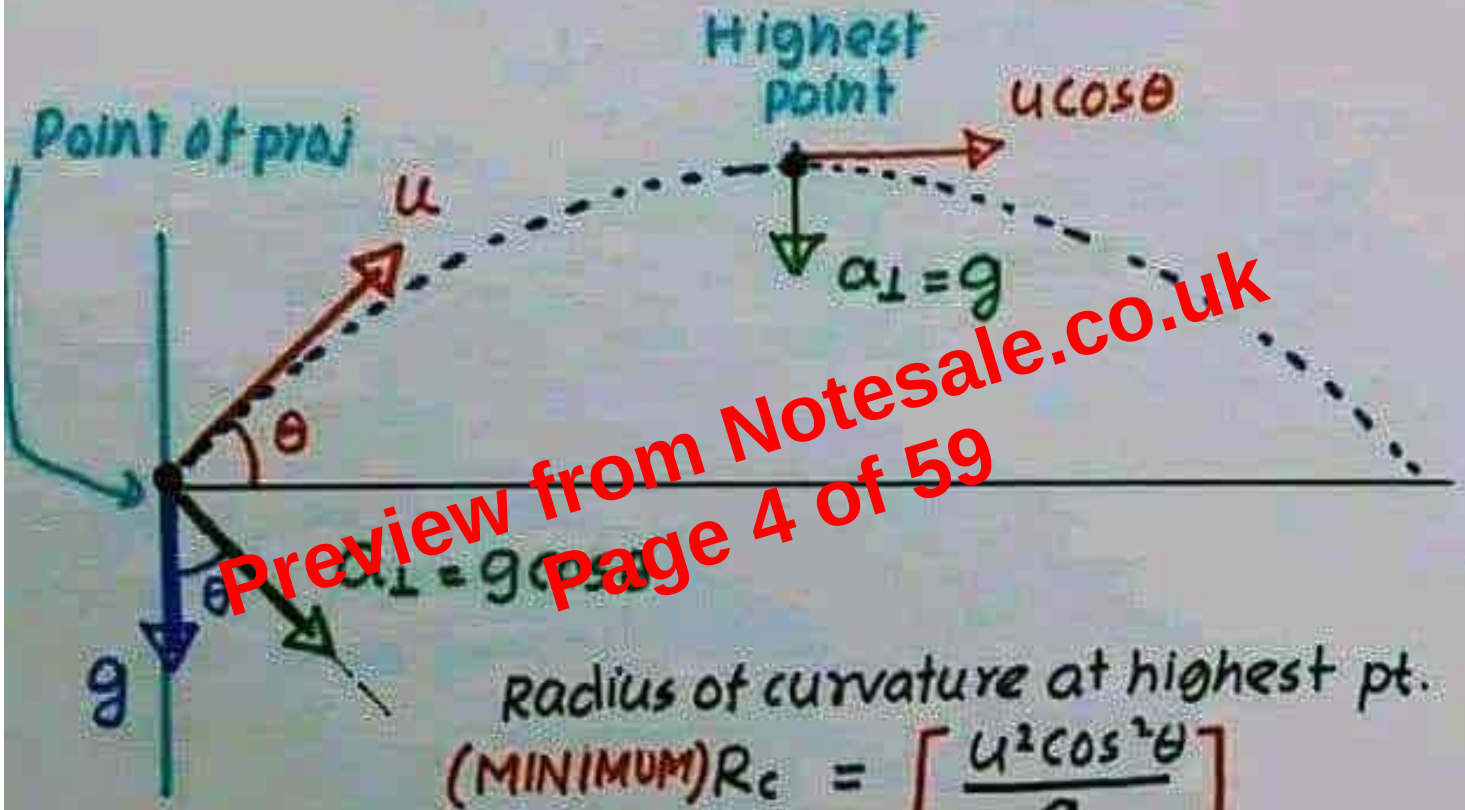
RADIUS OF CURVATURE OF TRAJECTORY

Radius of curvature (R_c)

$$= \frac{(\text{speed})^2}{a_\perp}$$

$a_\perp$ = acceleration perpendicular to instantaneous velocity



Radius of curvature at highest pt.

$$(\text{MINIMUM})\, R_c = \left[\frac{u^2\cos^2\theta}{g}\right]$$

Radius of curvature at point of projection

$$(\text{MAXIMUM})\, R_c = \left[\frac{u^2}{g\cos\theta}\right]$$

* Remembe R_c is min at highest point and max at point of projection

$$\frac{(R_c)_{max}}{(R_c)_{min}} = \frac{1}{\cos^3\theta} = \sec^3\theta$$

CHANGE IN MOMENTUM IN PROJECTILE

constant vertically downwards force

= weight = mg

acts on a projectile of mass (m) till it moves in air

t be the time of motion

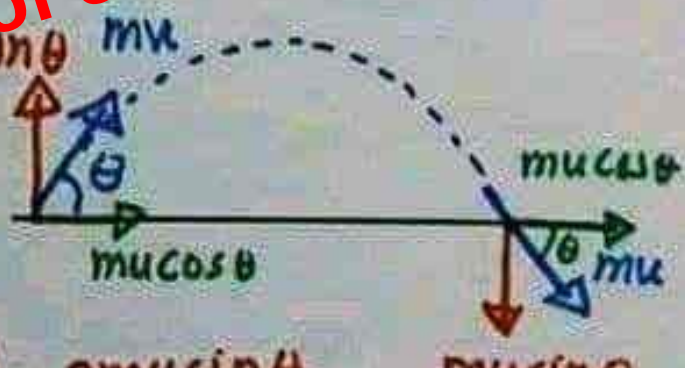
Apply impulse-momentum

$$|\Delta\vec{p}| = F \times t = mgt$$

Qu: A body of mass m is projected with u at an angle θ to horizontal. Find the magnitude of change in momentum till it strikes the ground / during time of flight

Hor. momentum remains unchanged

Vert. momentum gets reversed



$\Rightarrow \Delta p = 2mu\sin\theta$

OR $\Delta p = mg\left(\frac{2u\sin\theta}{g}\right) = 2mu\sin\theta$

* If $\theta = 45°$, $\Delta p = \sqrt{2}mu$ as $\sin\theta = \frac{1}{\sqrt{2}}$

TRY: A body of mass 500 gram is projected with $20ms^{-1}$ at an angle 60° to the horizontal. Find magnitude of change in momentum during first 1s of motion

(Hint: $\Delta p = mg \times t$) (answer: $5\ kg\,ms^{-1}$)

MAX DISTANCE A MAN CAN THROW

Let u be the maximum velocity with which a man can throw irrespective of direction

To throw upto max. height he will choose $\theta = 90°$ or project upwards

$$H_{max} = \frac{u^2}{2g}$$

To throw upto max. horizontal distance $\theta = 45°$

$$R_{max} = \frac{u^2}{g} \quad \left[\sin 2\theta = \sin 90° = 1\right]$$

Remember $\boxed{R_{max} = 2H_{max}}$

TRY: A man can throw a ball upto maximum height 30m. To what maximum horizontal distance he can throw the ball ?

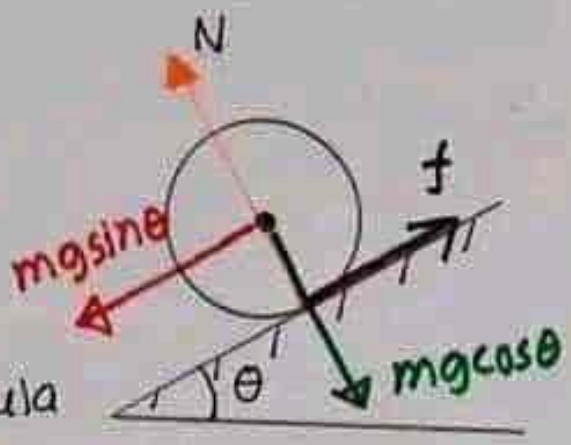
$$H_{max} = \frac{u^2}{2g}, \quad R_{max} = \frac{u^2}{g}$$

$$R_{max} = 2H_{max} = 2 \times 30$$

$$= 60m \text{ (ans)}$$

135. Round object rolling down a rough inclined plane (θ)

$$a = \frac{g\sin\theta}{1 + \frac{K^2}{r^2}}$$



*Note: The ratio infront of mr^2 in MI formula is $\left(\frac{K^2}{r^2}\right)$

example: ring $I = mr^2 = \left(\frac{1}{1}\right)mr^2$

disc $I = \left(\frac{1}{2}\right)mr^2$, solid sp $I = \left(\frac{2}{5}\right)mr^2$

sp. shell, $I = \left(\frac{2}{3}\right)mr^2$

greater $\frac{K^2}{r^2}$ ⇒ less a ⇒ greater time to roll down

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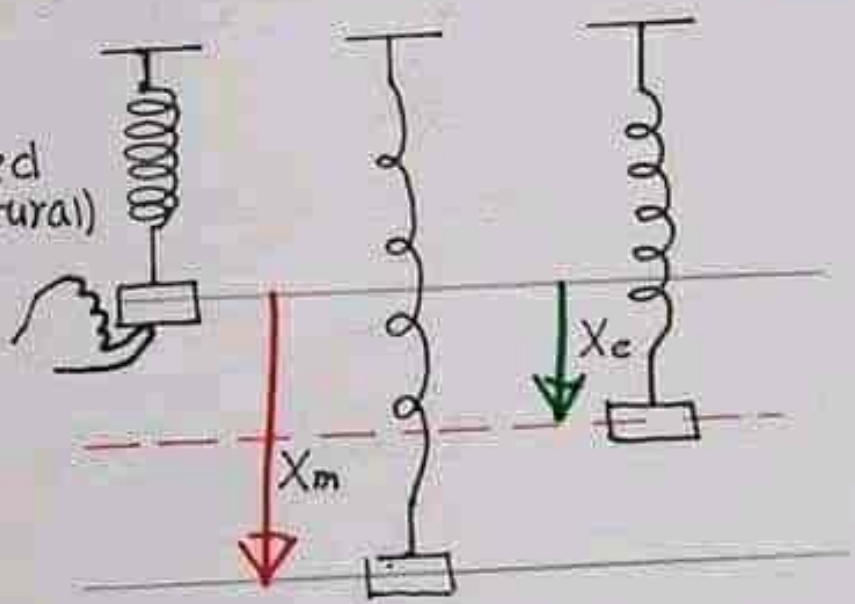
136. Equilibrium and maximum deformation in spring (X_e and X_m)



$$X_m = \frac{2mg}{K}$$

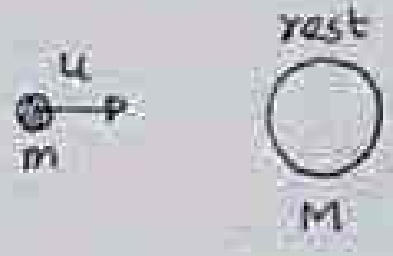
$$X_e = \frac{mg}{K}$$

$X_m = 2X_e = 2A$

A = oscillation amplitude

119. Small object projected towards havier at rest
ELASTIC COLLISION

Initial KE = K



Fraction of K retained by havier $f_H = \frac{4Mm}{(M+m)^2}$ = lost by lighter

Fraction of K retained by lighter

$$f_L = \left(\frac{M-m}{M+m}\right)^2$$ = lost by havier

commonly asked : neutron bombarded towards a nucleus (A) at rest collides elastically

Fraction retained by nucleus (havier) $= \frac{4A}{(A+1)^2}$ putting $M = mA$

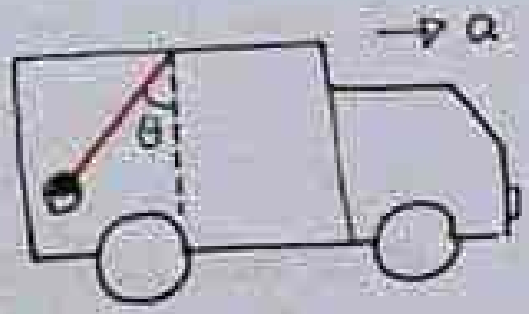
Fraction retained by neutron (lighter) $= \left(\frac{A-1}{A+1}\right)^2$

120. Pendulum inside horizontally accelerating vehicle

$\tan\theta = \frac{a}{g}$

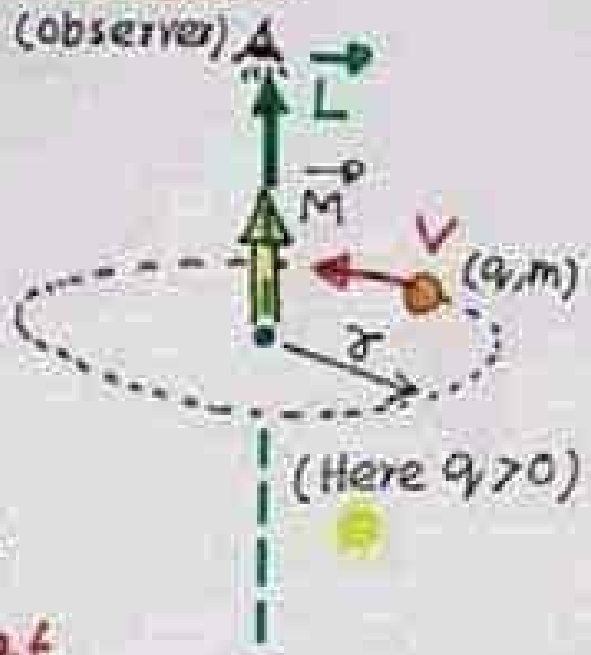
$T = m\sqrt{g^2+a^2}$

$g_{eff} = \sqrt{g^2+a^2}$



Charge in uniform circular motion

charged particle (q, m) moving along circle of radius r with constant speed v



Time period, $T = \frac{2\pi r}{v}$

frequency, $f = \frac{1}{T} = \frac{v}{2\pi r}$

- Equivalent current $I = qf$
  $= qv/2\pi r$
  (single turn circular loop)

- Magnetic moment $\vec{M} = NIA$
  $= \frac{qv}{2\pi r} \cdot \pi r^2$

$$|\vec{M}| = M = \frac{1}{2}qvr$$

q>0, towards
q<0, away from observer

- Angular momentum $\vec{L}$
  (always towards observer who can see anticlockwise circulation as m>0 always)

$$|\vec{L}| = mvr$$

- Gyromagnetic ratio

$$\frac{\vec{M}}{\vec{L}} = \frac{q}{2m}$$

$\vec{M} \parallel \vec{L}$ for $q>0$
$\vec{M} \nparallel \vec{L}$ for $q<0$

Question : 2

$\vec{A}$ and $\vec{B}$ are two vectors at an angle 120° such that resultant of these vectors is perpendicular to the smaller vector $\vec{A}$ and has magnitude 10 unit. Find the magnitudes of $\vec{A}$ and $\vec{B}$

solution:

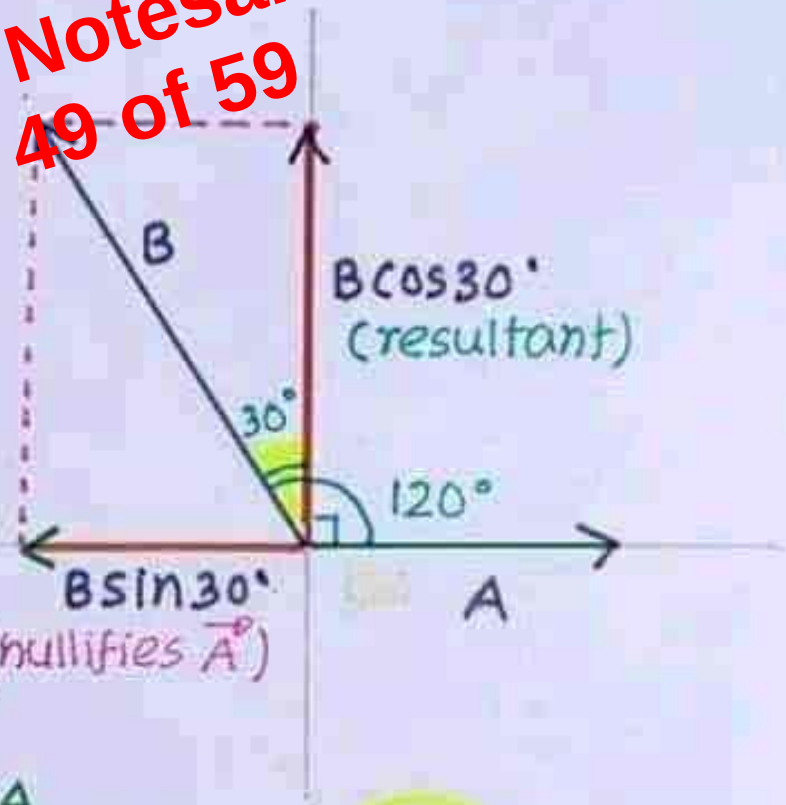
The arrangements of vectors is similar to RIVER-BOAT problem, crossing the river in or along shortest path (where $\vec{V_b}$ and $\vec{V_w}$ gives resultant $\vec{V_R}$ which must be absolutely transverse)

As shown:

$B\cos 30° = R$ ---(1)

$B\sin 30° = A$ ---(2)

Given $R = 10$



From (1):

$B \cdot \frac{\sqrt{3}}{2} = 10$

$\Rightarrow B = \frac{20}{\sqrt{3}}$ ✓

From (2): $B\sin 30° = A$

$\Rightarrow A = B \times \frac{1}{2} = \frac{20}{\sqrt{3}} \times \frac{1}{2} = \frac{10}{\sqrt{3}}$ ✓

Two COMMONLY ASKED Questions related to VECTORS

Question 1:

A man walks 20m towards East then 30m towards North and finally $30\sqrt{2}$m South-West find the magnitude and direction of total displacement.

solution:

To solve this question students usually show N-S-E-W directions properly and then start drawing vectors / displacements in sequence / one after another

DON'T DO THAT

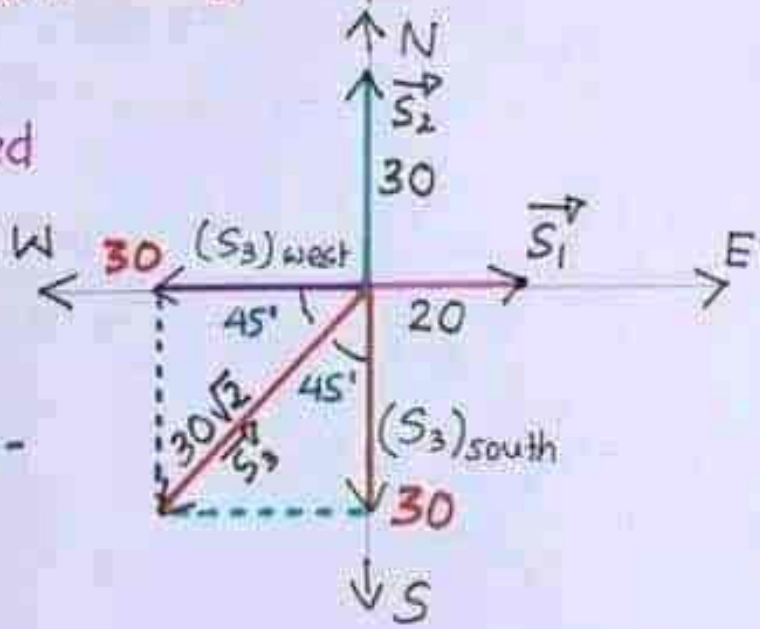
SIMPLE APPROACH :
- Show the directions
- Draw the vectors from origin (tails at origin / common point)
- Resolve the third displacement (which is oblique) into components Find resultant using components.

S_2 (north) and $(S_3)_{south}$ both 30m gets cancelled

resultant of S_1 (East) 20m and $(S_3)_{west}$ 30m

= 10m West

(Resultant displace-ment)



Pseudo Force

Ground is considered as master reference frame

Inertial frame :

wrt the ground acceleration of frame

$a_f = 0$

the frame is called inertial

Inertial frame may be at rest ($\vec{v} = 0$)
or may be moving uniformly
($\vec{v}$ = constant)

Example : An observer (a man) sitting at rest on a chair placed on the ground (v = 0)
An observer (a man) sitting on a chair inside a bus moving with constant velocity (v = c)

If the dynamics / statics of a system / body / block is analysed wrt inertial frame ONLY REAL FORCES are to be shown in the FBD (free body diagram)

✓ Real forces like weight, Tension normal reaction, friction