Properties of G.P:-

(i) a, b, c are in G.P $\Leftrightarrow b^2 = ac$

If $a_1, a_2, a_3, \ldots\ldots a_n$ are in GP

a) $a_n, a_{n-1}, a_{n-2}, \ldots\ldots\ldots a_1$ are in GP

b) $\lambda a_1, \lambda a_2, \lambda a_3, \ldots\ldots\ldots, \lambda a_n$ are in GP

$(\lambda \in R - \{0\})$

c) $a_1^n, a_2^n, a_3^n, \ldots\ldots a_n^n$ are in GP for $n \in R$

d) $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \frac{1}{a_n}$ are in GP

(ii) In a finite GP, the product of the terms equidistant from the beginning and end is always same and is equal to the product of the first and last terms.

(i.e) $a_2 a_{n-1} = a_3 a_{n-2} = a_4 a_{n-3} \ldots\ldots = a_1 a_n$

(iii) $a_1 . a_2 . a_3 \ldots\ldots a_n$ =(middle term)^n if n is odd

=(Product of two middle terms)$^{n/2}$ If n is even

(iv) If $a_1, a_2, a_3 \ldots a_n$ is a G.P of non zero, non negative terms then $\log a_1, \log a_2, \log a_3 \ldots\ldots \log a_n$ are in AP and vice versa

Geometric mean (GM):-

(i) The geometric mean G of any two numbers a and b is given by $\sqrt{ab}$ where a, G, b are in GP

(ii) If $a_1, a_2, a_3, \ldots\ldots a_n$ be n numbers then geometric mean of these numbers is

$$(a_1 . a_2 . a_3 \ldots . a_n)^{\frac{1}{n}}$$

(iii) The n numbers $G_1, G_2, G_3, \ldots\ldots G_n$ are said to be geometric means between a and b

If $a, G_1, G_2, G_3, \ldots\ldots\ldots G_n$, b are in GP

Here a = First term ; $b = (n+2)th$ term

$$\text{then } r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} ; \; G_1 = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$$G_2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}} \ldots\ldots ; \; G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

$$G_1 G_2 G_3 \ldots\ldots G_n = \left(\sqrt{ab}\right)^n = (GM \text{ of } a, b)^n$$

Some facts about GP :-

If $a_1, a_2, a_3, \ldots . a_n$ and $b_1, b_2, b_3, \ldots . b_n$ are two GP's then

(i) $a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3 \ldots\ldots a_n \pm b_n$ are not in GP

(ii) $a_1 b_1, a_2 b_2, a_3 b_3, \ldots\ldots\ldots a_n b_n$ and

$\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3} \ldots\ldots\ldots \frac{a_n}{b_n}$ are in GP

Increasing and decreasing GP:-

Let a, ar, ar^2, ... be a GP

a) If $a > 0; r > 1$ then it is increasing GP

b) If $a > 0;\; 0 < r < 1$ then it is decreasing GP

c) If $a < 0; r > 1$ then it is decreasing GP

d) If $a < 0; 0 < r < 1$ then it is increasing GP

Arithmetic - Geometric progression (AGP):

If $a_1, a_2, a_3 \ldots\ldots\ldots\ldots a_n$ are in AP and $b_1, b_2, b_3 \ldots\ldots\ldots\ldots b_n$ are in GP

then $a_1 b_1, a_2 b_2, a_3 b_3 \ldots\ldots\ldots\ldots a_n b_n$ is said to be AGP

An AGP is of the form

$$ab, (a+d)br, (a+2d)br^2, (a+3d)br^3, \ldots\ldots$$

(i) n^{th} term of an AGP

$$T_n = [a + (n-1)d]br^{n-1}$$

(i i)

$$S_n = \frac{ab}{1-r} + \frac{dbr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]br^n}{1-r} [r \neq 1]$$

(iii) If $-1 < r < 1$, then sum to infinite terms

$$s_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2}$$

Harmonic Progression (H.P):

A sequence is an H.P if the reciprocals of its terms form an AP.

H.P is of the form

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d} \ldots\ldots \frac{1}{a+(n-1)d}$$